**MECHANICS AND PROPERTIES OF MATTER**

The knowledge and understanding content for this unit is given below.

**Vectors**

1. Distinguish between distance and displacement.
2. Distinguish between speed and velocity.
3. Define and classify vector and scalar quantities.
4. Use scale diagrams, or otherwise, to find the magnitude and direction of the resultant of a number of displacements or velocities.
5. State what is meant by the resultant of a number of forces.
6. Carry out calculations to find the rectangular components of a vector.
7. Use scale diagrams, or otherwise, to find the magnitude and direction of the resultant of a number of forces.

**Equations of motion**

1. State that acceleration is the change in velocity per unit time.
2. Describe the principles of a method for measuring acceleration.
3. Draw an acceleration-time graph using information obtained from a velocity-time graph for motion with a constant acceleration.
4. Use the terms "constant velocity" and "constant acceleration" to describe motion represented in graphical or tabular form.
5. Show how the following relationships can be derived from basic definitions in kinematics:

$$v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as$$

6. Carry out calculations using the above kinematic relationships.

**Newton's Second Law, energy and power**

1. Define the newton.
2. Carry out calculations using the relationship $F = ma$ in situations where resolution of forces is not required.
3. Use free body diagrams to analyse the forces on an object.
4. Carry out calculations involving work done, potential energy, kinetic energy and power.

### Momentum and impulse

1. State that momentum is the product of mass and velocity.
2. State that the law of conservation of linear momentum can be applied to the interaction of two objects moving in one dimension, in the absence of net external forces.
3. State that an elastic collision is one in which both momentum and kinetic energy are conserved.
4. State that an inelastic collision is one in which only momentum is conserved.
5. Carry out calculations concerned with collisions in which the objects move in only one dimension.
6. Carry out calculations concerned with explosions in one dimension.
7. Apply the law of conservation of momentum to the interaction of two objects moving in one direction to show that:
   a) the changes in momentum of each object are equal in size and opposite in direction
   b) the forces acting on each object are equal in size and opposite indirection.
8. State that impulse = force × time.
9. State that impulse = change in momentum.
10. Carry out calculations using the relationship, impulse = change of momentum.

### Density and Pressure

1. State that density is mass per unit volume.
2. Carry out calculations involving density, mass and volume.
3. Describe the principles of a method for measuring the density of air.
4. State and explain the relative magnitudes of the densities of solids, liquids and gases.
5. State that pressure is the force per unit area, when the force acts normal to the surface.
6. State that one pascal is one newton per square metre.
7. Carry out calculations involving pressure, force and area.
8. State that the pressure at a point in a fluid at rest is given by h⑧g.
9. Carry out calculations involving pressure, density and depth.
10. Explain buoyancy force (upthrust) in terms of the pressure difference between the top and bottom of an object.

### Gas laws

1. Describe how the kinetic model accounts for the pressure of a gas.
2. State that the pressure of a fixed mass of gas at constant temperature is inversely proportional to its volume.
3. State that the pressure of a fixed mass of gas at constant volume is directly proportional to its temperature measured in kelvin K.
4. State that the volume of a fixed mass of gas at constant pressure is directly proportional to its temperature measured in kelvin K.
5. Carry out calculations to convert temperatures in $^{o}$C to K and vice versa.
6. Carry out calculations involving pressure, volume and temperature of a fixed mass of gas using the general gas equation.
7. Explain what is meant by absolute zero of temperature.
8. Explain the pressure-volume, pressure-temperature and volume-temperature laws qualitatively in terms of the kinetic model.

## VECTORS

### Distance and Displacement.

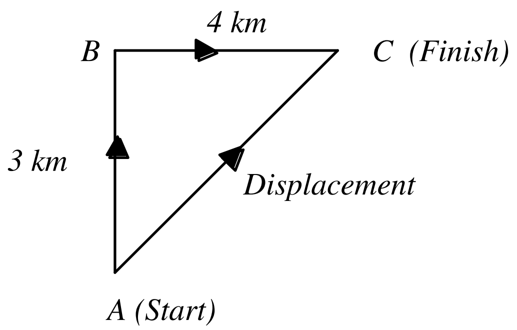
Distance is the total path length. It is fully described by magnitude (size) alone. Displacement is the direct length from a starting point to a finishing point. To describe displacement **both** magnitude **and** direction must be given.

*Example*

*A woman walks 3 km due North (000) and then 4 km due East (090).*

*Find her a) distance travelled*

*b) displacement i.e. how far she is from where she started ?*




*Using a scale of 1cm: 1km draw an accurate scale diagram as shown above.*

*a) Distance travelled = AB + BC = 7 km*

*b) Measuring AC = 5 cm.*

*Convert using the scale gives the magnitude of the displacement = 5 km*

*Use a protractor to check angle BAC = $53^o$ that is $53^o$ east of north.*

*Displacement = 5 km (053)*

### Speed and Velocity

These two quantities are fundamentally different.

$$\text{Average speed} = \frac{\text{distance}}{\text{time}} \qquad \text{Average velocity} = \frac{\text{displacement}}{\text{time}}$$

Velocity has an associated direction, being the same as that of the displacement. The unit for both these quantities is metres per second, m $s^{-1}$.

### Vectors and Scalars

A **scalar** quantity is completely defined by stating its **magnitude**.
A **vector** quantity is completely defined by stating its **magnitude** and **direction**.
Examples are given below.

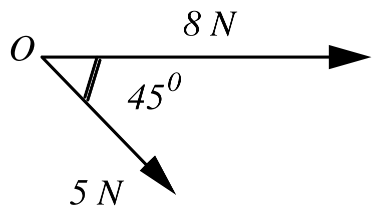
| Vectors | Scalars |
|---|---|
| Displacement | Distance |
| Velocity | Speed |
| Acceleration | Time |
| Force | Mass |
| Impulse | Energy |

**Addition of Vectors**

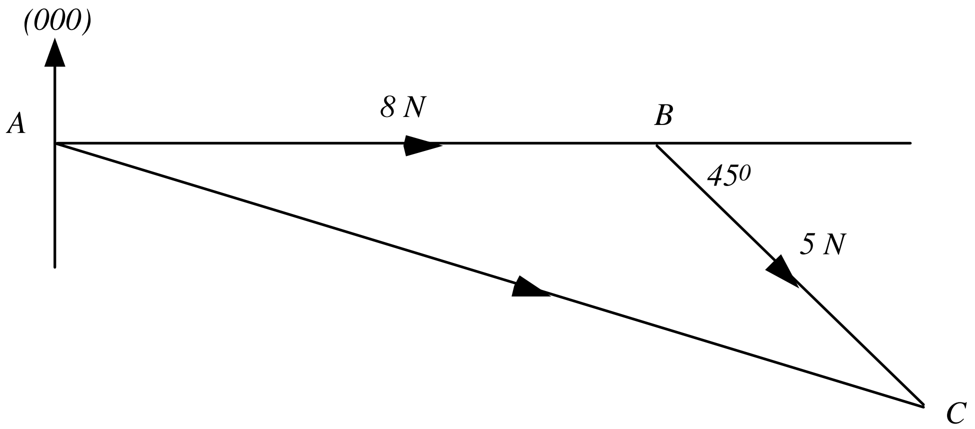
When vectors are being added, their magnitude and direction must be taken into account. This can be done using a scale diagram and adding the vectors 'tip to tail', then joining the starting and finishing points. The final sum is known as the resultant, the single vector that has the same effect as the sum of the individuals.

***Example***

Find the resultant force acting at point O.




Step 1: Choose a suitable scale, e.g. 1 cm to 1 N.
Step 2: Arrange arrows "tip to tail".
Step 3: Draw in resultant vector, measuring its length and direction.




| | | | |
|---|---|---|---|
| 1 cm to 1 N. | AC | = | 12.2 cm |
| | Force | = | 12.2 N |
| | | | Using a protractor, angle BAC measures $12^0$ |
| | Bearing | = | $90^0 + 12^0 = 102^0$ |
| | Resultant Force | = | 12.2 N at (102) |

**Vectors at right angles**

If the vectors are at right angles then it may be easier to use Pythagoras to find the resultant and trigonometry to find an angle.

**Addition of more than two vectors**

Use a scale diagram and ensure that each vector is placed "tip to tail" to the previous vector. The resultant vector is the vector from the starting point to the finishing point in magnitude and direction.

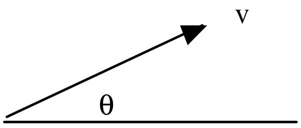
**Resultant of a number of forces**

The resultant of a number of forces is that single force which has the same effect, in both magnitude and direction, as the sum of the individual forces.

**Rectangular components of a vector**

***Resolution of a vector into horizontal and vertical components.***

Any vector v can be split up into a horizontal component $v_h$ and vertical component $v_v$.

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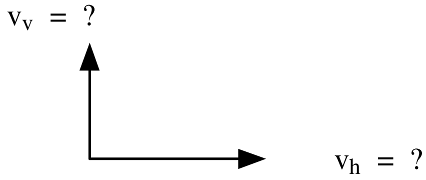
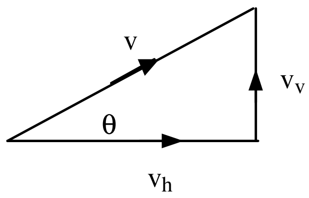
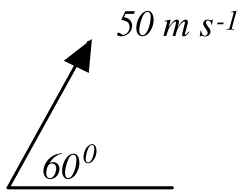



$$\sin\theta = \frac{v_v}{v} \qquad \boxed{\mathbf{v_v = v\ \sin\theta}}$$

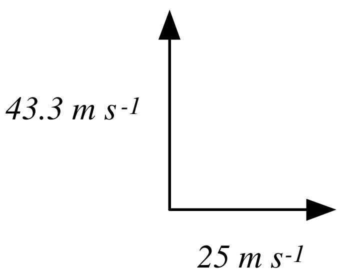
$$\cos\theta = \frac{v_h}{v} \qquad \boxed{\mathbf{v_h = v\ \cos\theta}}$$

*Example*
*A shell is fired from a cannon as shown.*
*Calculate its a) horizontal component of velocity*
*b) vertical component of velocity.*




a) $v_h = v \cos u = 50 \cos 60° = 25$ m s$^{-1}$
b) $v_v = v \sin u = 50 \sin 60° = 43$ m s$^{-1}$

So

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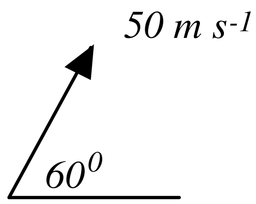


## EQUATIONS OF MOTION

### Acceleration

Acceleration is defined as the change in velocity per unit time.
The unit is metre per second squared, m $s^{-2}$.

$$\boxed{a = \frac{v - u}{t}}$$

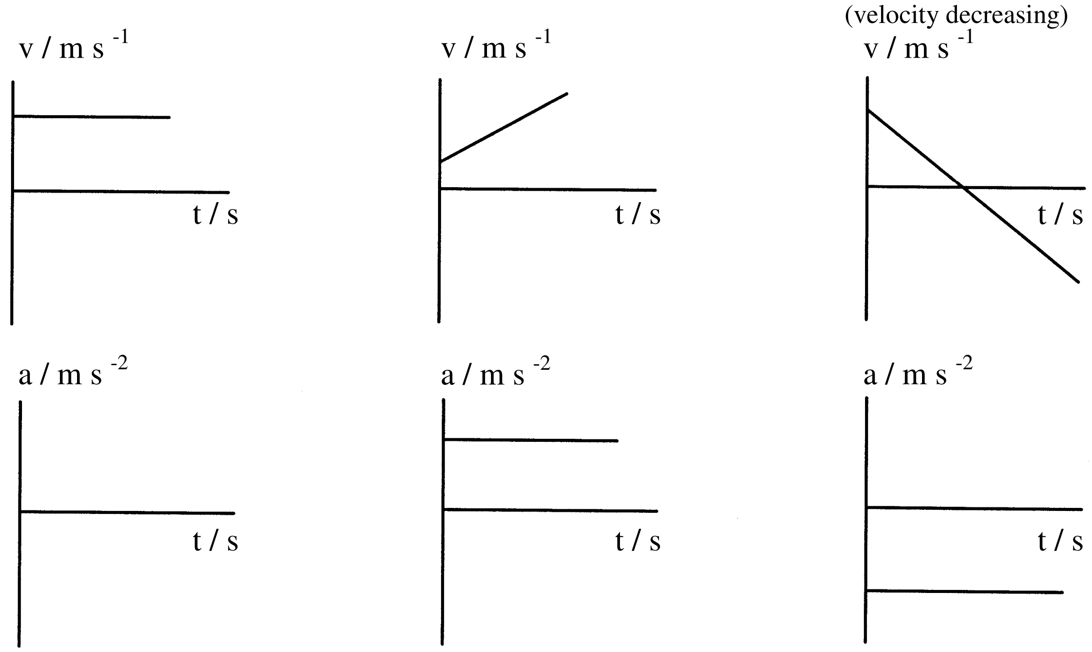
where v = final velocity
u = initial velocity
t = time taken

### Measuring acceleration

Acceleration is measured by determining the initial velocity, final velocity and time taken. A double mask which interrupts a light gate can provide the data to a microcomputer and give a direct reading of acceleration.

### Acceleration-time and velocity-time graphs

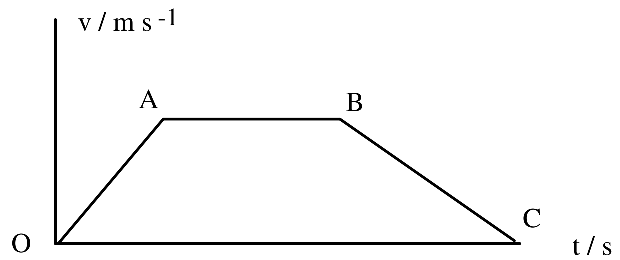
Constant velocity | Constant positive acceleration (velocity increasing) | Constant deceleration Constant negative acceleration (velocity decreasing)




a = 0 | acceleration is positive | acceleration is negative

### Constant velocity and constant acceleration

The velocity time graph below illustrates these terms.




OA is constant acceleration, the acceleration is positive.
AB is constant velocity, the acceleration is zero.
BC is constant deceleration, the acceleration is negative.

**Equations of motion**

$$\boxed{v = u + at}$$

where: u - initial velocity of object at time t = 0
v - final velocity of object at time t
a - acceleration of object
t - time to accelerate from u to v
s - displacement in time t.

$$\boxed{s = ut + \frac{1}{2}at^2}$$

$$\boxed{v^2 = u^2 + 2as}$$

These equations of motion apply providing:

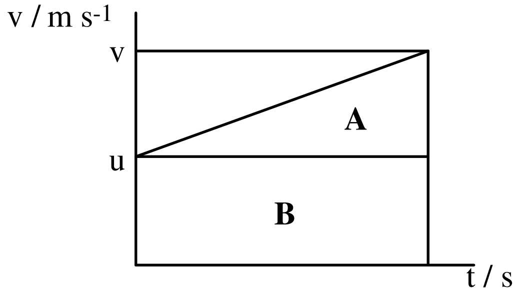
- the motion is in a straight line
- the acceleration is uniform.

When using the equations of motion, **note**:

- the quantities u, v, s and a are all vector quantities
- a positive direction must be chosen and quantities in the reverse direction must be given a negative sign
- a deceleration will be negative, for movement in the positive direction.

**Derivation of equations of motion**

The velocity - time graph for an object accelerating uniformly from u to v in time t is shown below.




$$a = \frac{v - u}{t}$$

Changing the subject of the formula gives:

$$\boxed{v = u + at} \qquad \text{---------} \quad [1]$$

The displacement s in time t is equal to the area under the velocity time graph.

$$\text{Area} = \text{Area of triangle A + area of rectangle B}$$

$$s = \frac{1}{2}(v-u)t + ut$$

but from equation 1, $v-u = at$

$$s = \frac{1}{2}(at)t + ut$$

$$\boxed{s = ut + \frac{1}{2}at^2} \qquad \text{---------} \quad [2]$$

Using v = u + at,

$$v^2 = (u + at)^2$$

$$v^2 = u^2 + 2uat + a^2t^2$$

$$v^2 = u^2 + 2a(ut + \frac{1}{2}at^2)$$

Since $s = ut + \frac{1}{2}at^2$

$$\boxed{v^2 = u^2 + 2as} \qquad \text{---------} \quad [3]$$

**Projectile motion**

A projectile has a combination of vertical and horizontal motions. Various experiments show that these horizontal and vertical motions are totally independent of each other.
Closer study gives the following information about each component.

**Horizontal:** **constant speed**

**Vertical:** **constant acceleration downward (due to gravity).**

*Example*

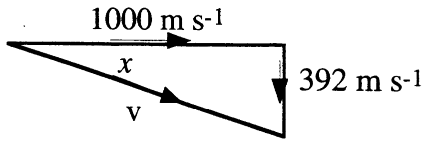
*An object is released from an aircraft travelling horizontally at 1000 m s$^{-1}$. The object takes 40 s to reach the ground.*

*a) What is the horizontal distance travelled by the object?*
*b) What was the height of the aircraft when the object was released?*
*c) Calculate the vertical velocity of the object just before impact.*
*d) Find the resultant velocity of the object just before hitting the ground.*

*Before attempting the solution, you should divide your page into horizontal and vertical and enter appropriate information given or known.*

| ***Horizontal*** | ***Vertical*** |
|---|---|
| $v_h$ = 1000 m s$^{-1}$ $\quad t$ = 40 s | $t$ = 40 s $\quad u_v = 0 \quad a$ = 9.8 m s$^{-2}$ |
| *a)* $s_h$ = ? | b) $s_v$ = ? $\quad s_v = ut + \frac{1}{2}at^2$ |
| $s_h$ = v x t = 1000 x 40 = 40000 m | = 0 + $\frac{1}{2}$ x 9.8 x $40^2$ |
| | = 7840 m |
| | *c)* $v_v$ = ? |
| | $v_v = u + at$ = 0 + 9.8 x 40 |
| | = 392 m |
| | $v_v$ = 392 m s$^{-1}$ (downwards) |

*d)*




$$\tan x = \frac{392}{1000} \qquad x = 21^o$$

$$v^2 = 1000^2 + 392^2$$
$$v = 1074 \text{ m s}^{-1}$$

*Resultant velocity = 1074 m s$^{-1}$ at (111)*

## NEWTON'S SECOND LAW, ENERGY AND POWER

Dynamics deals with the forces causing motion and the properties of the resulting moving system.

### Newton's 1st Law of Motion

Newton's 1st law of Motion states that an object will remain at rest or travel with a constant speed in a straight line (constant velocity) unless acted on by an unbalanced force.

### Newton's 2nd Law

Newton's 2nd law of motion states that the acceleration of an object:

- varies directly as the unbalanced force applied if the mass is constant
- varies inversely as the mass if the unbalanced force is constant.

These can be combined to give

$$a \propto \frac{F}{m}$$

$$a = \frac{kF}{m} \quad \text{where k is a constant}$$

$$kF = ma$$

The unit of force, the **newton** is defined as the resultant force which will cause a mass of 1kg to have an acceleration of 1 m $s^{-2}$. Substituting in the above equation.

$$k \times 1 = 1 \times 1$$
$$k = 1$$

Provided F is measured in newtons, the equation below applies.

$$\boxed{F = ma} \text{ — m s}^{-2}$$

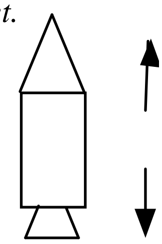
N kg

### Free Body Diagrams

Some examples will have more than one force acting on an object. It is advisable to draw a diagram of the situation showing the direction of all forces present acting through one point. These are known as free body diagrams.

*Examples*

1. *On take off, the thrust on a rocket of mass 8000 kg is 200,000 N. Find the acceleration of the rocket.*



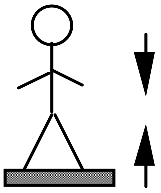
*Thrust = 200,000 N*

*Weight = mg = 8000 × 9.8 = 78,400 N*

*Resultant force = 200000 - 78,400 = 121,600 N*

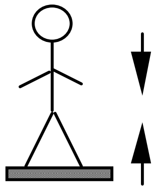
$$a = \frac{F}{m} = \frac{121600}{8000} = 15.2 \text{ m s}^{-2}$$

2. *A woman is standing on a set of bathroom scales in a stationary lift (a normal everyday occurrence!). The reading on the scales is 500 N. When she presses the ground floor button, the lift accelerates downwards and the reading on the scales at this moment is 450 N. Find the acceleration of the lift.*



Weight = 500 N

Force upwards = 500 N



W = 500 N

F = 450 N
(reading on scales)

Lift is stationary, forces balance

W = F

= 500 N

Lift accelerates downwards, unbalanced force acts.

Resultant Force = Weight - Force from floor

= W - F

= 500 – 450

= 50 N

$$a = \frac{\text{Resultant Force}}{m}$$

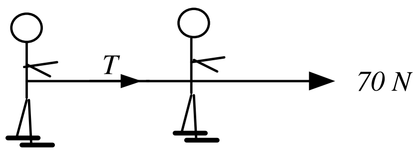
$$= \frac{50}{50}$$

$$= 1 \text{ m s}^{-2}$$

3. *Tension*

*A ski tow pulls 2 skiers who are connected by a thin nylon rope along a frictionless surface. The tow uses a force of 70 N and the skiers have masses of 60 kg and 80 kg. Find a) the acceleration of the system*

*b) the tension in the rope.*




60 kg 80kg

*a)* *Total mass, m = 140 kg*

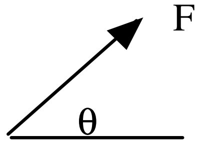
$$a = \frac{F}{m} = \frac{70}{140} = 0.5 \text{ m s}^{-2}$$

*b)* *Consider the 60 kg skier alone.*

*Tension, T = ma = 60 × 0.5 = 30 N*

## Resolution of a Force

In the previous section, a vector was split into horizontal and vertical components. This can obviously apply to a force.

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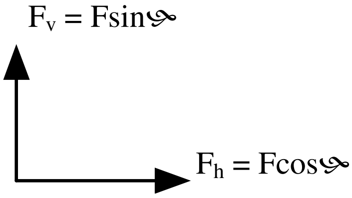
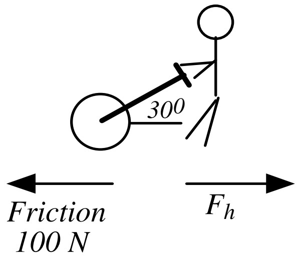



*Example*

*A man pulls a garden roller of mass 100 kg with a force of 200 N acting at $30^0$ to the horizontal. If there is a frictional force of 100 N between the roller and the ground, what is the acceleration of the roller along the ground?*



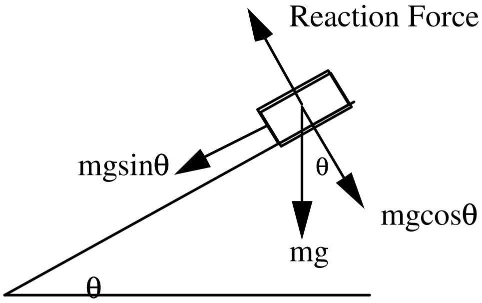

$F_h = F\cos\theta = 200\cos 30^0 = 173.2\ N$

Resultant $F_h = 173.2 - \text{Friction} = 173.2 - 100 = 73.2\ N$

$$a = \frac{F}{m} = \frac{73.2}{100} = 0.732\ m\ s^{-2}$$

## Force Acting Down a Plane

If an object is placed on a slope then its weight acts vertically downwards. A certain component of this force will act down the slope. The weight can be split into two components at right angles to each other.

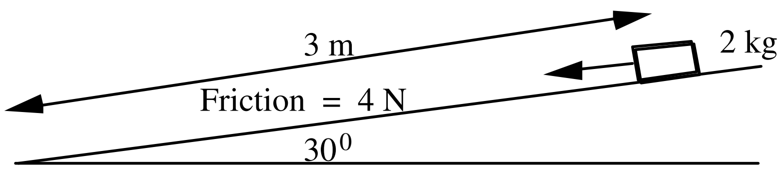



| Component of weight down slope = mgsinθ |
|---|

| Component perpendicular to slope = mgcosθ |
|---|

*Example*

*A wooden block of mass 2 kg is placed on a slope at 30° to the horizontal as shown. A frictional force of 4 N acts up the slope. The block slides down the slope for a distance of 3 m. Determine the speed of the block at the bottom of the slope.*




*Component of weight acting down slope = mg sin30° = 2 × 9.8 × 0.5 = 9.8 N*

Resultant force down slope = 9.8 - friction = 9.8 - 4 = 5.8 N

$a = F/$
$= 5.8 / 2$
$= 2.9\ m\ s^{-2}$

$v^2 = u^2 + 2as$
$= 0 + 2 \times 2.9 \times 3$
$= 17.4$
$v = 4.2\ m\ s^{-1}$

**Conservation of Energy**

The total energy of a closed system must be conserved, although the energy may change its form.

The equations for calculating **kinetic energy** $E_k$, **gravitational potential energy** $E_p$ and **work done** are given below.

| | | |
|---|---|---|
| $E_p = mgh$ | $E_k = \frac{1}{2}mv^2$ | work done = force × displacement |

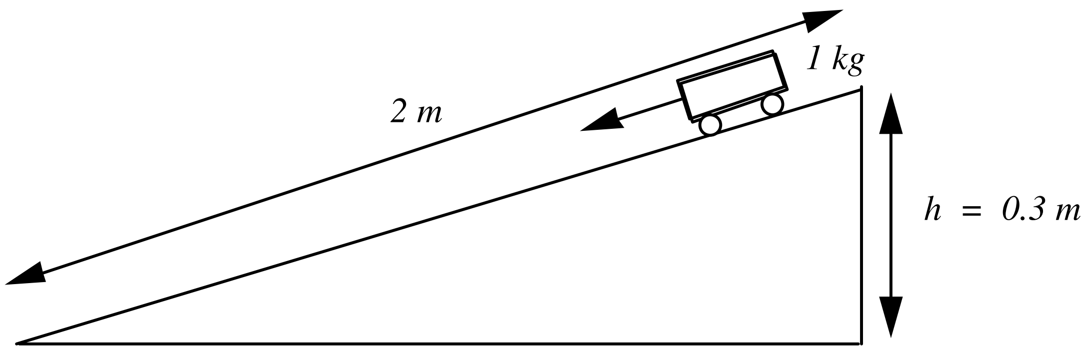
Energy and work are measured in joules **J**.

*Example*

*A trolley is released down a slope from a height of 0.3 m. If its speed at the bottom is found to be 2 m s$^{-1}$, find a) the energy difference between the $E_p$ at top and $E_k$ at the bottom.*

*b) the work done by friction*

*c) the force of friction on the trolley*




a) $E_p$ *at top* $= mgh = 1 \times 9.8 \times 0.3 = 2.94$ *J*

$E_k$ *at bottom* $= \frac{1}{2}mv^2 = \frac{1}{2} \times 1 \times 4 = 2$ *J*

*Energy difference = 0.94 J*

b) Work done by friction = energy difference (due to heat, sound) = 0.94 J

c) Work done = Force of friction × d = 0.94 J d = 2 m

$$F = \frac{0.94}{2} = 0.47 \text{ N}$$

Force of friction = 0.47 N

***Power***

*Power is the rate of transformation of energy from one form to another.*

$$P = \frac{\text{energy}}{\text{time}} = \frac{\text{work done}}{\text{time}} = \frac{F \times \text{displacement}}{t} = F \times \text{average velocity}$$

Power is measured in watts W.

## MOMENTUM AND IMPULSE

The momentum of an object is given by:

Momentum = mass × velocity of the object.

$$\boxed{\text{Momentum} = mv}$$

kg m $s^{-1}$ kg m $s^{-1}$

**Note:** momentum is a vector quantity.
The **direction** of the momentum is the same as that of the velocity.

### Conservation of Momentum

When two objects collide it can be shown that momentum is conserved **provided** there are no external forces applied to the system.
For any collision:

$$\boxed{\text{Total momentum of all objects before = total momentum of all objects after.}}$$

***Elastic and inelastic collisions***

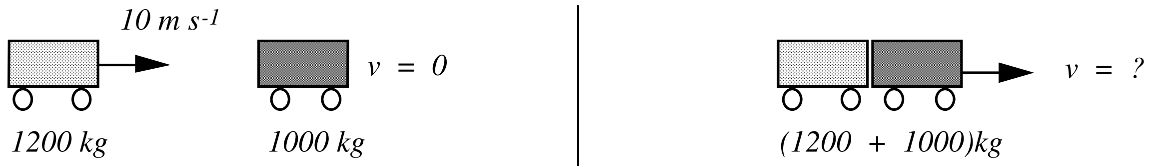
An elastic collision is one in which **both** kinetic energy and momentum are conserved.
An inelastic collision is one in which **only** momentum is conserved.

*Example*

*a)* *A car of mass 1200 kg travelling at 10 m $s^{-1}$ collides with a stationary car of mass 1000 kg. If the cars lock together find their combined speed.*

*b)* By comparing the kinetic energy before and after the collision, find out if the collision is elastic or inelastic.

Draw a simple sketch of the cars before and after the collision.




BEFORE AFTER

*a)*

*Momentum* $= mv$
$= 1200 \times 10$
$= 12000$ kg m $s^{-1}$

*Momentum* $= mv$
$= (1200 + 1000)\,v$
$= 2200v$ kg m $s^{-1}$

*Total momentum before = Total momentum after*

$$12000 = 2200v$$

$$\frac{12000}{2200} = v$$

$$v = 5.5 \text{ m s}^{-1}$$

*b)* $Ek = \frac{1}{2}mv^2$ $\qquad$ $Ek = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 1200 \times 10^2$ $\qquad$ $= \frac{1}{2} \times 2200 \times 5.5^2$

$= 60{,}000\ J$ $\qquad$ $= 33{,}275\ J$

*Kinetic energy is not the same, so the collision is* ***inelastic***

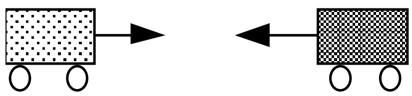
***Vector nature of momentum***

Remember momentum is a vector quantity, so direction is important. Since the collisions dealt with will act along the same line, then the directions can be simplified by giving:

**momentum to the right a positive sign and**
**momentum to the left a negative sign.**

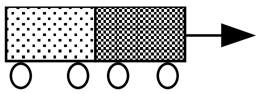
*Example*
*Find the unknown velocity below.*

*v = 4m s*$^{-1}$



*8 kg* $\qquad$ *6 kg*

*BEFORE*
Momentum = mv
= (8 × 4) - (6 × 2)
= 20 kg m s$^{-1}$



*v = ?*

*(8 + 6 )kg*

*AFTER*
Momentum = mv
= (8 + 6)v
= 14v

*Total momentum before = Total momentum after*
*20 = 14v*

$$v = \frac{20}{14} = 1.43 \text{ m s}^{-1}$$

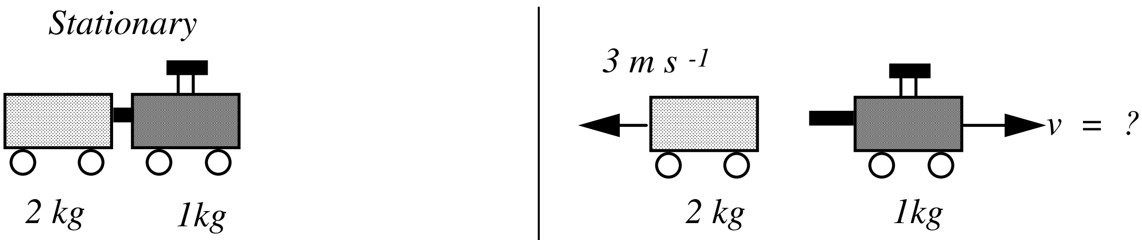
Trolleys will move to the right at 1.43 m s$^{-1}$ since v is positive.

**Explosions**

A single stationary object may explode into two parts. The total initial momentum will be zero. Hence the total final momentum will be zero. Notice that the kinetic energy increases in such a process.

*Example*

*Two trolleys shown below are exploded apart. Find the unknown velocity.*




| BEFORE | AFTER |
|---|---|
| *Total momentum = mv* | *Total momentum = mv* |
| *= 0* | *= - (2x3) + 1v* |
| | *= - 6 + v* |

*Total momentum before = Total momentum after*

$0 = -6 + v$

$v = 6$ m s$^{-1}$ to the right (since v is positive).

**Impulse**

An object is accelerated by a force F for a time, t. The unbalanced force is given by:

$$F = ma = \frac{m(v - u)}{t} = \frac{mv - mu}{t}$$

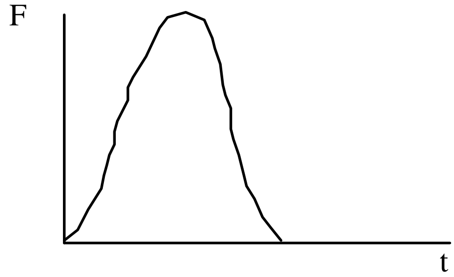
$$\text{Unbalanced force} = \frac{\text{change in momentum}}{\text{time}} = \text{rate of change of momentum}$$

$$\boxed{Ft = mv - mu}$$

The term Ft is called the **impulse** and is equal to the change in momentum.

**Note:** the unit of impulse, Ns will be equivalent to kg m s$^{-1}$.

The concept of **impulse** is useful in situations where the force is not constant and acts for a very short period of time. One example of this is when a golf ball is hit by a club. During contact the unbalanced force between the club and the ball varies with time as shown below.




Since F is not constant the impulse (Ft) is equal to the **area** under the graph. In any calculation involving impulse the unbalanced force calculated is always the average force and the maximum force experienced would be greater than the calculated average value.

*Examples*

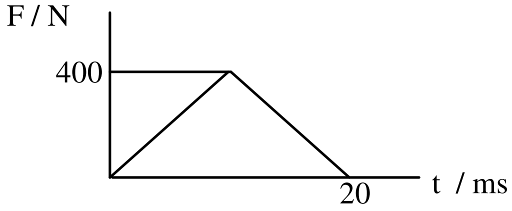
1. *In a snooker game, the cue ball, of mass 0.2 kg, is accelerated from the rest to a velocity of 2 m $s^{-1}$ by a force from the cue which lasts 50 ms. What size of force is exerted by the cue?*

   $u = 0$ $v = 2$ m $s^{-1}$ $t = 50$ ms $= 0.05$ s $m = 0.2$ kg $F = ?$

   $Ft = mv - mu$

   $F \times 0.05 = 0.2 \times 2$ $F = 8$ N

2. *A tennis ball of mass 100 g, initially at rest, is hit by a racket. The racket is in contact* with *the ball for 20 ms and the force of contact varies over this period as shown in the graph. Determine the speed of the ball as it leaves the racket.*




*Impulse = Area under graph*

$= \frac{1}{2} \times 20 \times 10^{-3} \times 400 = 4$ N s

$u = 0$ $m = 100$ g $= 0.1$ kg $v = ?$

$Ft = mv - mu = 0.1v$

$4 = 0.1v$

$v = 40$ m $s^{-1}$

3. *A tennis ball of mass 0.1 kg travelling horizontally at 10 m $s^{-1}$ is struck in the opposite direction by a tennis racket. The tennis ball rebounds horizontally at 15 m $s^{-1}$ and is in contact with the racket for 50 ms. Calculate the force exerted on the ball by the racket.*

   $m = 0.1$ kg $u = 10$ m $s^{-1}$ $v = -15$ m $s^{-1}$ *(opposite direction to u)*

   $t = 50$ ms $= 0.05$ s

   $Ft = mv - mu$

   $0.05F = 0.1 \times (-15) - 0.1 \times 10$

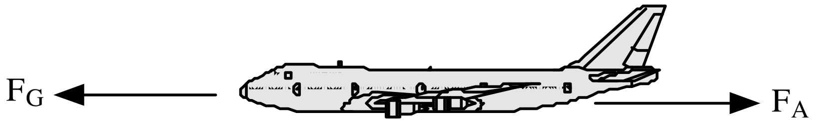
   $= -1.5 - 1 = -2.5$

   $F = \frac{-2.5}{0.05} = -50$ N *(Negative indicates force in opposite direction to initial velocity)*

**Newton's 3rd Law and Momentum**

Newton's 3rd law states that if an object A exerts a force (ACTION) on object B then object B will exert an equal and opposite force (REACTION) on object A.
This law can be proved using the conservation of momentum.
Consider a jet engine expelling gases in an aircraft.

Let $F_A$ be the force on the aircraft by the gases
and $F_G$ be the force on the gases by the engine (aircraft).




Let the positive direction be to the **left** (direction of Fa)

In a small time, let $m_G$ be the mass of the gas expelled and
$m_A$ be the mass of the aircraft.

total momentum before = total momentum after

$$O = m_G v_G + m_A v_A$$

$$m_G v_G = - m_A v_A \text{ [} v_B \text{ and } v_A \text{ in opposite directions]}$$

$$(m_G v_G - O) = -(m_A v_A - O)$$

Change in momentum of gas = - (change in momentum of aircraft)

Changes in momentum of each object are equal in size but opposite in direction.

If forces act in time, t

$$\text{Force} = \frac{\text{change in momentum}}{t}$$

$$F_G = \frac{(m_G \ v_G - O)}{t} \qquad FA = \frac{(m_A v_A - O)}{t} \qquad (u_A = u_G = O)$$

But $(m_G \ v_G - O) = (m_A v_A - O)$ from above

$F_G = - F_A$ since t is the same for the engine and gas.

**The forces acting are equal in size and opposite in direction.**

## DENSITY AND PRESSURE

### Density

The mass per unit volume of a substance is called the density, r.
(The symbol, ⑧, is the Greek letter rho).

⑧ = $\frac{m}{V}$

⑧ = density in kilograms per cubic metre, kg m$^{-3}$

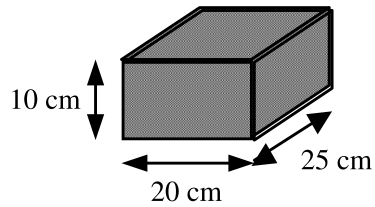
m = mass in kilograms, kg
V = volume in cubic metres, m$^3$

*Example*
Calculate the density of a 10 kg block of carbon measuring 10 cm by 20 cm by 25 cm.

First, calculate volume, V, in m$^3$ : V = 0.1 × 0.2 × 0.25 = 0.005 m$^3$

*⑧ = ?*
*m = 10 kg*
*V = 0.005 m$^3$*

*⑧ =* $\frac{m}{V}$ $\frac{10}{0.005}$
*= 2000 kg m$^{-3}$*



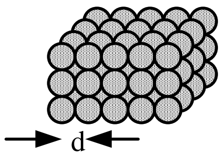

### Densities of Solids, Liquids and Gases

From the table opposite, it can be seen that the relative magnitude of the densities of solids and liquids are similar but the relative magnitude of gases are smaller by a factor of 1000.
When a solid melts to a liquid, there is little relative change in volume due to expansion. The densities of liquids and solids have similar magnitudes.
When a liquid evaporates to a gas, there is a large relative change in volume due to the expansion of the material. The volume of a gas is approximately 1000 times greater than the volume of the same mass of the solid or liquid form of the substance.
The densities of gases are smaller than the densities of solids and liquids by a factor of approximately 1000.

| Substance | Density (kg m$^{-3}$) |
|---|---|
| Ice | 920 |
| Water | 1000 |
| Steam | 0.9 |
| Aluminium | 2700 |
| Iron | 7860 |
| Perspex | 1190 |
| Ethanol | 791 |
| Olive oil | 915 |
| Vinegar | 1050 |
| Oxygen | 1.43 |
| Nitrogen | 1.25 |

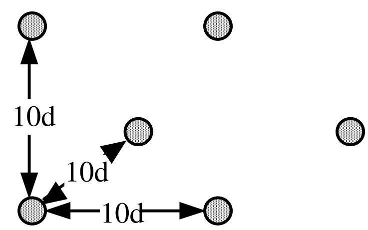
It follows, therefore, that the spacing of the particles is a gas must be approximately 10 times greater than in a liquid or solid.

**Solid or liquid**




Volume occupied by each particle
= d$^3$

**Gas**




Volume occupied by each particle
= (10d)$^3$ = 1000 d$^3$

## Pressure

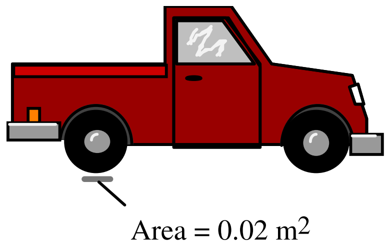
Pressure on a surface is defined as the force acting normal (perpendicular) to the surface.

$$\boxed{p = \frac{F}{A}}$$

p = pressure in pascals, Pa
F = normal force in newtons, N
A = area in square metres, $m^2$

1 pascal is equivalent to 1 newton per square metre; ie 1 Pa = 1 N $m^{-2}$.

*Example*
*Calculate the pressure exerted on the ground by a truck of mass 1600 kg if each wheel has an area of 0.02 $m^2$ in contact* with the ground.




*Total area A = 4 × 0.02 = 0.08 $m^2$*
Normal force F = weight of truck = mg = 1600 × 9.8 = 15680 N

*p = ?*
*F = 15680 N*
*A = 0.08 m2*

$$p = \frac{F}{A} = \frac{15680}{0.08}$$

*= 196,000 Pa or 196 kPa*

## Pressure In Fluids

Fluid is a general term which describes liquids and gases. Any equations that apply to liquids at rest equally apply to gases at rest.
The pressure at a point in a fluid at rest of density ρ, depth h below the surface, is given by

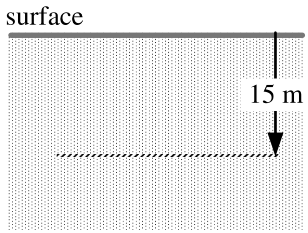
$$\boxed{p = h \rho g}$$

p = pressure in pascals, Pa
h = depth in metres, m
ρ = density of the fluid in kg $m^{-3}$
g = gravitational field strength in N $kg^{-1}$

*Example*
*Calculate the pressure due to the water at a depth of 15 m in water.*

*p = ?*
*h = 15 m*
$\rho_{water}$ *= 1000 kg $m^{-3}$*
g = 9.8 N kg-1

*p = h ρ g*
*= 15 × 1000 × 9.8*
*= 147000 Pa*

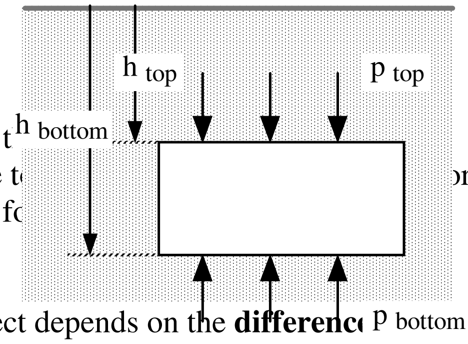


### Buoyancy Force (Upthrust)

When a body is immersed in a fluid, it appears to "lose" weight. The body experiences an upwards force due to being immersed in the fluid. This upwards force is called an **upthrust.**

This upthrust or buoyancy force can be explained in terms of the forces acting on the body due to the pressure acting on each of the surfaces of the body.

Pressure on the top surface $p_{top} = h_{top}\rho g$
Pressure on bottom surface $p_{bottom} = h_{bottom}\rho g$




The bottom surface of the body is at a greater depth t... pressure on the bottom surface is greater than on the t... orce upwards on the body due to the liquid. This upward fc...

Notice that the buoyancy force (upthrust) on an object depends on the **differenc...** pressure on the top and bottom of the object. Hence the value of this buoyance force does **not** depend on the depth of the object under the surface.

## GAS LAWS

### Kinetic Theory of Gases

The kinetic theory tries to explain the behaviour of gases using a model. The model considers a gas to be composed of a large number of very small particles which are far apart and which move randomly at high speeds, colliding elastically with everything they meet.

***Volume*** The volume of a gas is taken as the volume of the container. The volume occupied by the gas particles themselves is considered so small as to be negligible.

***Temperature*** The temperature of a gas depends on the kinetic energy of the gas particles. The faster the particles move, the greater their kinetic energy and the higher the temperature.

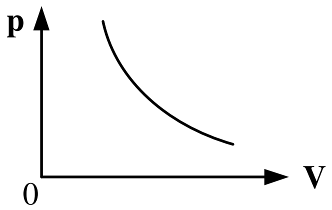
***Pressure*** The pressure of a gas is caused by the particles colliding with the walls of the container. The more frequent these collisions or the more violent these collisions, the greater will be the pressure.

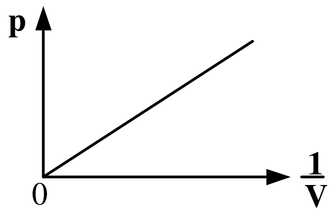
**Relationship Between Pressure and Volume of a Gas**

For a fixed mass of gas at a constant temperature, the pressure of a gas is inversely proportional to its volume.

| $p \propto \frac{1}{V}$ | $p \times V = \text{constant}$ | $p_1 V_1 = p_2 V_2$ |
|---|---|---|

*Graph*



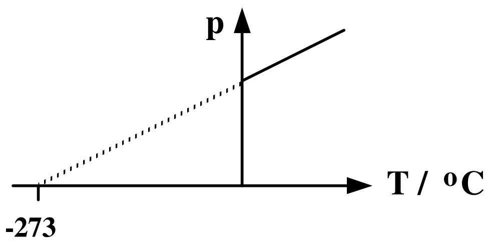




*Example*
*The pressure of a gas enclosed in a cylinder by a piston changes from 80 kPa to 200 kPa. If there is no change in temperature and the initial volume was 25 litres, calculate the new volume.*

| | |
|---|---|
| $p_1 = 80$ kPa | $p_1 V_1 = p_2 V_2$ |
| $V_1 = 25$ litres | $80 \times 25 = 200 \times V_2$ |
| $p_2 = 200$ kPa | $V_2 = 10$ litres |
| $V_2 = ?$ | |

**Relationship Between Pressure and Temperature of a Gas**

If a graph is drawn of pressure against temperature in degrees celsius for a fixed mass of gas at a constant volume, the graph is a straight line which does not pass through the origin. When the graph is extended until the pressure reaches zero, it crosses the temperature axis at -273 $^{o}$C. This is true for all gases.

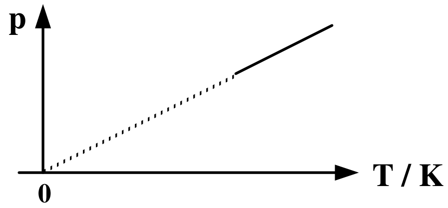



**Kelvin Temperature Scale**

-273$^{o}$C is called **absolute zero** and is the zero on the kelvin temperature scale. At a temperature of absolute zero, 0 K, all particle motion stops and this is therefore the lowest possible temperature.
One division on the kelvin temperature scale is the same size as one division on the celsius temperature scale, i.e. temperature **differences** are the same in kelvin as in degrees celsius, e.g. a temperature increase of 10°C is the same as a temperature increase of 10 K.
Note the unit of the kelvin scale is the kelvin, K, **not** degrees kelvin, °K!

**Converting Temperatures Between °C and K**

Converting °C to K add 273
Converting K to °C subtract 273
If the graph of pressure against temperature is drawn using the kelvin temperature scale, zero on the graph is the zero on the kelvin temperature scale and the graph now goes through the origin.




For a fixed mass of gas at a constant volume, the pressure of a gas is directly proportional to its temperature measured in kelvin (K).

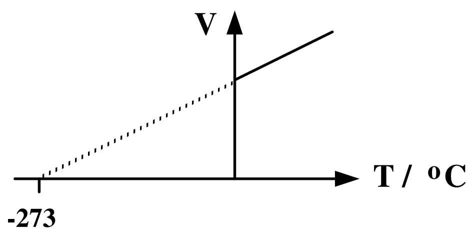
$$\boxed{p \propto T} \quad \boxed{\frac{p}{T} = \text{constant}} \quad \boxed{\frac{p_1}{T_1} = \frac{p_2}{T_2}}$$

*Example*
*Hydrogen in a sealed container at 27 °C has a pressure of 1.8 × 105 Pa. If it is heated to a temperature of 77 °C, what will be its new pressure?*

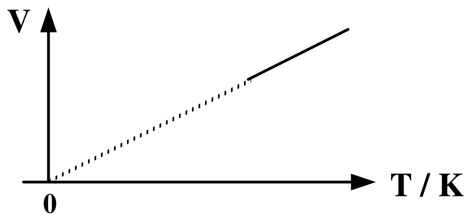
$p_1 = 1.8 \times 10^5$ *Pa*
$T_1 = 27$ *°C* $= 300$ *K*
$p_2 = ?$
$T_2 = 77$ °C $= 350$ K  $p_2 = 2.1 \times 10^5$ Pa

**Relationship Between Volume and Temperature of a Gas**

If a graph is drawn of volume against temperature, in degrees celsius, for a fixed mass of gas at a constant pressure, the graph is a straight line which does not pass through the origin. When the graph is extended until the volume reaches zero, again it crosses the temperature axis at -273 °C. This is true for all gases.




If the graph of volume against temperature is drawn using the kelvin temperature scale, the graph now goes through the origin.




For a fixed mass of gas at a constant pressure, the volume of a gas is directly proportional to its temperature measured in kelvin (K).

$$V \} T \qquad \frac{V}{T} = \text{constant} \qquad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

*Example*
*400 cm$^3$ of air is at a temperature of 20 °C. At what temperature will the volume be 500 cm$^3$ if the air pressure does not change?*

*$V_1$ = 400 cm$^3$*
*$T_1$ = 20 °C = 293 K*
*$V_2$ = 500 cm$^3$*
*$T_2$ = ?*

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad \frac{400}{293} = \frac{500}{T_2}$$

*$T_2$ = 366 K = 93 °C (convert back to temperature scale in the question)*

**Combined Gas Equation**

By combining the above three relationships, the following relationship for the pressure, volume and temperature of a fixed mass of gas is true for all gases.

$$\frac{p \times V}{1} = \text{constant} \qquad \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

*Example*
*A balloon contains 1.5 m$^3$ of helium at a pressure of 100 kPa and at a temperature of 27 °C. If the pressure is increased to 250 kPa at a temperature of 127 °C, calculate the new volume of the balloon.*

*$p_1$ = 100 kPa*
$V_1$ = 1.5 m$^3$
$T_1$ = 27 °C = 300 K
$p_2$ = 250 kPa
$V_2$ = ?
$T_2$ = 127 °C = 400 K

$$\frac{100 \times 1.5}{300} = \frac{200 \times V_2}{400}$$

$$V_2 = 0.8 \text{ m}^3$$

**Gas Laws and the Kinetic Theory of Gases**

***Pressure - Volume (constant mass and temperature)***

Consider a volume V of gas at a pressure p. If the volume of the container is reduced without a change in temperature, the particles of the gas will hit the walls of the container more often (but not any harder as their average kinetic energy has not changed). This will produce a larger force on the container walls. The area of the container walls will also reduce with reduced volume.
As volume decreases, then the force increases and area decreases resulting in, from the definition of pressure, an increase in pressure,
i.e. volume decreases hence pressure increases and vice versa.

**Pressure - Temperature (constant mass and volume)**

Consider a gas at a pressure p and temperature T. If the temperature of the gas is increased, the kinetic energy and hence speed of the particles of the gas increases. The particles collide with the container walls more violently and more often. This will produce a larger force on the container walls.

As temperature increases, then the force increases resulting in, from the definition of pressure, an increase in pressure,

i.e. temperature increases hence pressure increases and vice versa.

***Volume - Temperature (constant mass and pressure)***

Consider a volume V of gas at a temperature T. If the temperature of the gas is increased, the kinetic energy and hence speed of the particles of the gas increases. If the volume was to remain constant, an increase in pressure would result as explained above. If the pressure is to remain constant, then the volume of the gas must increase to increase the area of the container walls that the increased force is acting on, i.e. volume decreases hence pressure increases and vice versa.