All questions should be attempted *Marks*

1. Calculate the length of the vector $2\boldsymbol{i} - 3\boldsymbol{j} + \sqrt{3}\boldsymbol{k}$. **(2)**

2. (*a*) Show that $(x - 3)$ is a factor of $f(x)$, where $f(x) = 2x^3 + 3x^2 - 23x - 12$. **(2)**

   (*b*) Hence express $f(x)$ in its fully factorised form. **(2)**

3. Find $\int (6x^2 - x + \cos x)\, dx$. **(4)**

4. Find the value of k for which the vectors $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 3 \\ k - 1 \end{pmatrix}$ are perpendicular. **(3)**

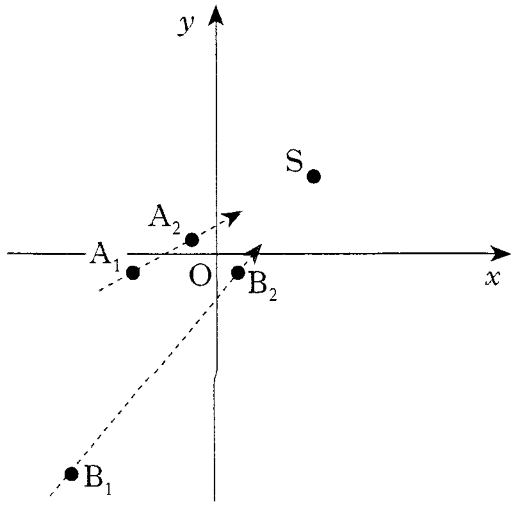
5. Find the equation of the median AD of triangle ABC where the coordinates of A, B and C are (–2, 3), (–3, –4) and (5, 2) respectively. **(3)**

[Turn over

6. A Royal Navy submarine, exercising in the Firth of Clyde, is stationary on the seabed below a point S on the surface. S is the point (5, 4) as shown in the diagram.

A radar operator observes the frigate "Achilles" sailing in a straight line, passing through the points A_1 (−4, −1) and A_2 (−1, 1).

Similarly, the frigate "Belligerent" is observed sailing in a straight line, passing through the points B_1 (−7,−11) and B_2 (1, −1).



If both frigates continue to sail in straight lines, will either or both frigates pass directly over the submarine? **(5)**

7. Find $\frac{dy}{dx}$ where $y = \frac{4}{x^2} + x\sqrt{x}$. **(4)**

8. Find the exact solutions of the equation

$$4\sin^2 x = 1, \quad 0 \leq x < 2\pi.$$ **(4)**

9. Find the equation of the circle which has P(−2, −1) and Q(4, 5) as the end points of a diameter. **(3)**

Marks

10. The point $P(-2, b)$ lies on the graph of the function $f(x) = 3x^3 - x^2 - 7x + 4$.

(*a*) Find the value of b. **(1)**

(*b*) Prove that this function is increasing at P. **(3)**

11. The functions f and g, defined on suitable domains, are given by

$f(x) = \dfrac{1}{x^2 - 4}$ and $g(x) = 2x + 1$.

(*a*) Find an expression for $h(x)$ where $h(x) = g(f(x))$. Give your answer as a single fraction. **(3)**

(*b*) State a suitable domain for h. **(1)**

12. Given that $\tan \alpha = \dfrac{\sqrt{11}}{3}$, $0 < \alpha < \dfrac{\pi}{2}$, find the exact value of $\sin 2\alpha$. **(3)**

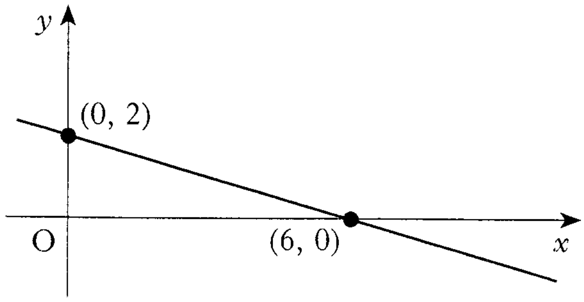
13. Solve the simultaneous equations

$$k \sin x° = 5$$

$$k \cos x° = 2 \quad \text{where } k > 0 \text{ and } 0 \leq x \leq 360.$$ **(4)**

[Turn over

14. The straight line shown in the diagram has equation $y = f(x)$.
Determine $f'(x)$. **(2)**

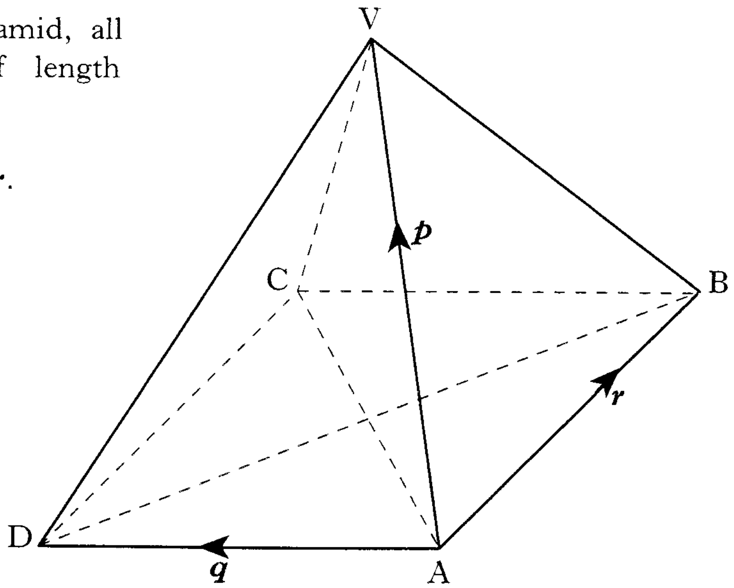


15. Solve algebraically the equation

$$\cos 2x° + \cos x° = 0, \qquad 0 \leq x < 360.$$ **(5)**

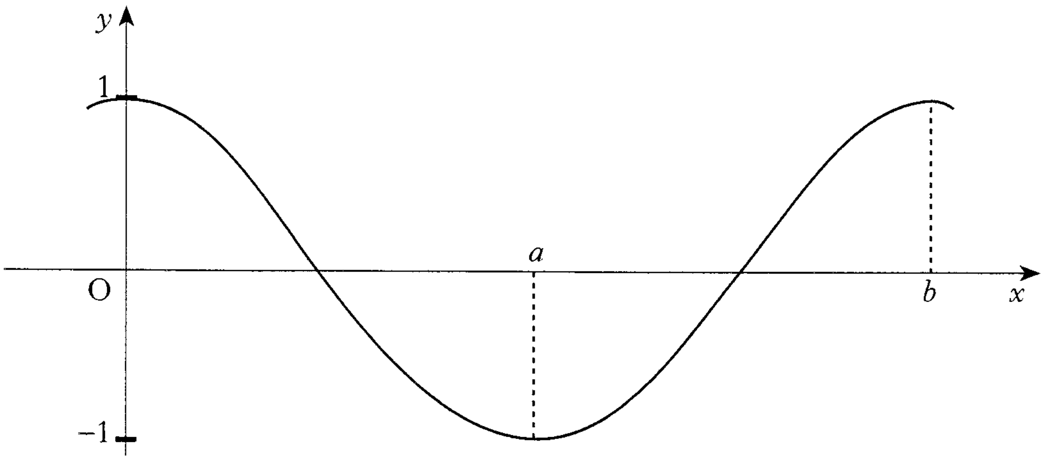
16. In the square-based pyramid, all the eight edges are of length 3 units.

$\overrightarrow{AV} = \boldsymbol{p}$, $\overrightarrow{AD} = \boldsymbol{q}$, $\overrightarrow{AB} = \boldsymbol{r}$.
Evaluate $\boldsymbol{p}.(\boldsymbol{q} + \boldsymbol{r})$. **(4)**



Marks

17. Part of the graph of $y = f(x)$ is shown in the diagram. This graph has stationary points at $x = 0$, $x = a$ and $x = b$.



(a) Sketch the graph of $y = f'(x)$ for $0 \leq x \leq b$. **(3)**

(b) If $a = \pi$ and $b = 2\pi$, write down a possible expression for $f'(x)$. **(1)**

18. The amount A grams of a radioactive substance after a time t minutes is given by $A = A_0e^{-kt}$ where A_0 is the initial amount of the substance and k is a constant.

In 3 minutes, 10 grams of the substance Bismuth are reduced to 9 grams through radioactive decay.

(a) Find the value of k. **(3)**

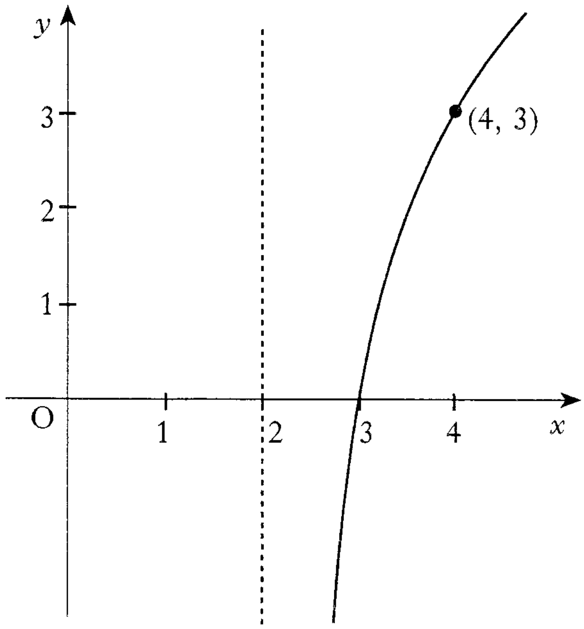
The half-life of a substance is the length of time in which half the substance decays.

(b) Find the half-life of Bismuth. **(2)**

[Turn over

Marks

19. The diagram shows a sketch of the graph of $y = f(x)$, where $f(x) = a\log_2(x - b)$.
Find the values of a and b. **(3)**



20. The roots of the equation $(x - 1)(x + k) = -4$ are equal.
Find the values of k. **(5)**

21. A ball is thrown vertically upwards. The height h metres of the ball t seconds after it is thrown, is given by the formula $h = 20t - 5t^2$.

(*a*) Find the speed of the ball when it is thrown (i.e. the rate of change of height with respect to time of the ball when it is thrown). **(3)**

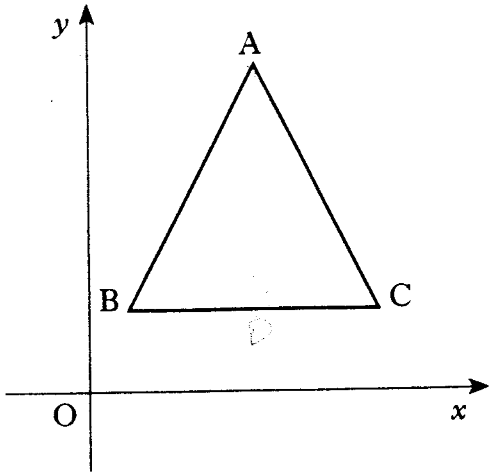
(*b*) Find the speed of the ball after 2 seconds.
Explain your answer in terms of the movement of the ball. **(2)**

[*END OF QUESTION PAPER*]

Higher - 1995 Paper II

All questions should be attempted — *Marks*

1. A triangle ABC has vertices A(4, 8), B(1, 2) and C(7, 2).

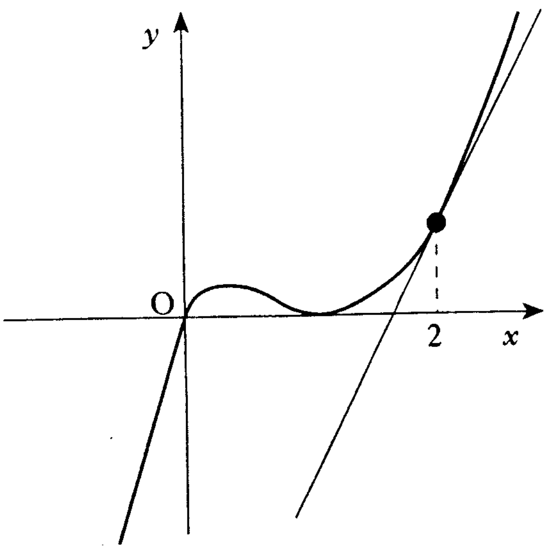


(*a*) Show that the triangle is isosceles. **(2)**

(*b*) (i) The altitudes AD and BE intersect at H, where D and E lie on BC and CA respectively. Find the coordinates of H. **(7)**

(ii) Hence show that H lies one quarter of the way up DA. **(1)**

2. The diagram shows a sketch of part of the graph of $y = x^3 - 2x^2 + x$.



(*a*) Show that the equation of the tangent to the curve at $x = 2$ is $y = 5x - 8$. **(4)**

(*b*) Find algebraically the coordinates of the point where this tangent meets the curve again. **(5)**

Marks

3. Trees are sprayed weekly with the pesticide, "Killpest", whose manufacturers claim it will destroy 65% of all pests. Between the weekly sprayings, it is estimated that 500 new pests invade the trees.

   A new pesticide, "Pestkill", comes onto the market. The manufacturers claim that it will destroy 85% of existing pests but it is estimated that 650 new pests per week will invade the trees.

   Which pesticide will be more effective in the long term? **(7)**

Marks

6. A system of 3 equations in 3 unknowns can be solved by a method known as Gaussian Elimination as shown below.

Example

Solve the system of equations by Gaussian Elimination.

$$\begin{aligned} x + 2y - 3z &= 11 \\ 2x + 2y - z &= 11 \\ 3x - 2y + 4z &= -4 \end{aligned}$$

A Write out the coefficients in an array:

- Row 1
- Row 2
- Row 3

$$\left.\begin{array}{rrr|r} 1 & 2 & -3 & 11 \\ 2 & 2 & -1 & 11 \\ 3 & -2 & 4 & -4 \end{array}\right.$$

B Keep Row 1 the same. Make Row 2 and Row 3 each begin with a zero by subtracting multiples of Row 1 from them.

- Row 1 is kept the same
- Row 2 becomes "Row 2 – 2 × Row 1"
- Row 3 becomes "Row 3 – 3 × Row 1"

$$\left.\begin{array}{rrr|r} 1 & 2 & -3 & 11 \\ \mathbf{0} & \mathbf{-2} & \mathbf{5} & \mathbf{-11} \\ \mathbf{0} & \mathbf{-8} & \mathbf{13} & \mathbf{-37} \end{array}\right.$$

C Keep Row 1 and Row 2 the same. Make Row 3 begin with two zeros, by subtracting a multiple of Row 2 from it.

- Row 1 is kept the same
- Row 2 is kept the same
- Row 3 becomes "Row 3 – 4 × Row 2"

$$\left.\begin{array}{rrr|rl} 1 & 2 & -3 & 11 & \text{..........(1)} \\ 0 & -2 & 5 & -11 & \text{..........(2)} \\ \mathbf{0} & \mathbf{0} & \mathbf{-7} & \mathbf{7} & \text{..........(3)} \end{array}\right.$$

D

- Line (3) gives $\quad -7z = 7, \quad z = -1$
- Line (2) gives $\quad -2y + 5z = -11$
  $\quad -2y + (-5) = -11, \quad y = 3$
- Line (1) gives $\quad x + 2y - 3z = 11$
  $\quad x + 6 + 3 = 11, \quad x = 2$

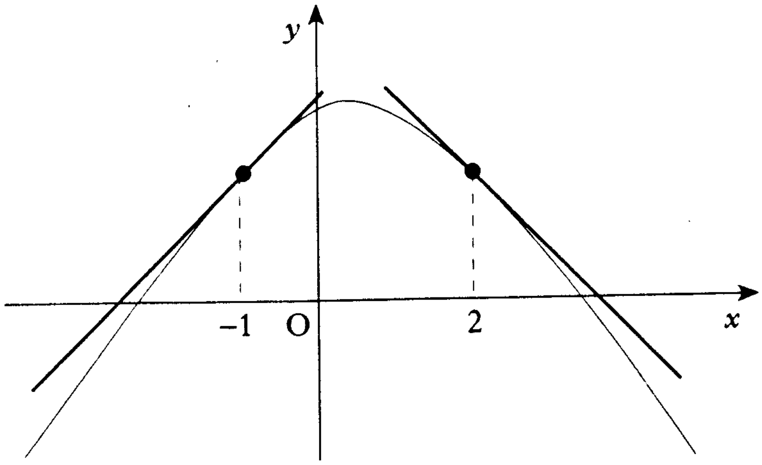
So the solution is $x = 2$, $y = 3$, $z = -1$

Solve the following system of equations by Gaussian Elimination **as shown above**.

$$\begin{aligned} x - 2y + z &= 6 \\ 3x + y - z &= 7 \\ 4x - y + 2z &= 15 \end{aligned}$$

(7)

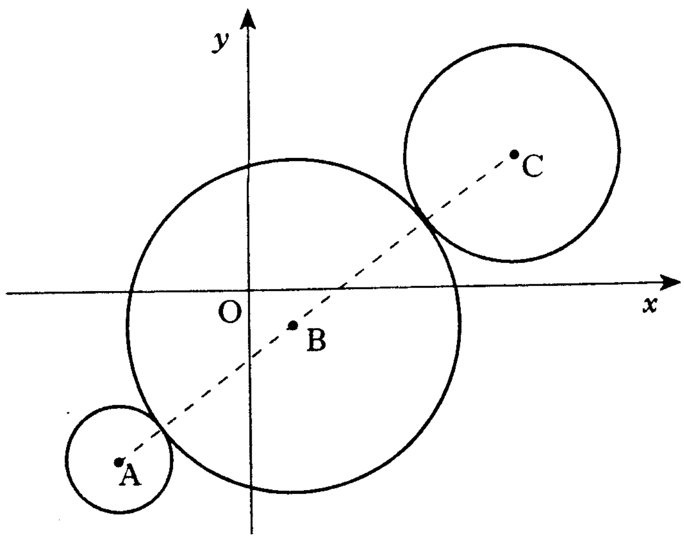
7. The parabola $y = ax^2 + bx + c$ crosses the y-axis at (0, 3) and has two tangents drawn, as shown in the diagram.



The tangent at $x = -1$ makes an angle of 45° with the positive direction of the x-axis and the tangent at $x = 2$ makes an angle of 135° with the positive direction of the x-axis.

Find the values of a, b and c. **(8)**

8. When newspapers were printed by lithograph, the newsprint had to run over three rollers, illustrated in the diagram by three circles. The centres A, B and C of the three circles are collinear.



The equations of the circumferences of the outer circles are

$(x + 12)^2 + (y + 15)^2 = 25$ and $(x - 24)^2 + (y - 12)^2 = 100$.

Find the equation of the central circle. **(8)**

Marks

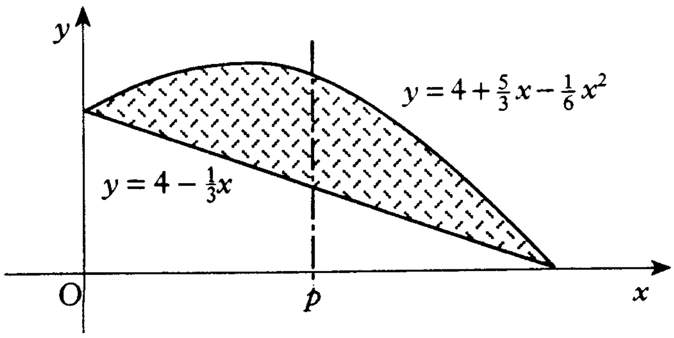
9. (*a*) By writing $\sin 3x$ as $\sin(2x + x)$, show that
$\sin 3x = 3\sin x - 4\sin^3 x$. **(4)**

(*b*) Hence find $\int \sin^3 x\, dx$. **(4)**

10. When building a road beside a vertical rockface, engineers often use wire mesh to cover the rockface. This helps to prevent rocks and debris from falling onto the road. The shaded region of the diagram below represents a part of such a rockface.

This shaded region is bounded by a parabola and a straight line.

The equation of the parabola is $y = 4 + \frac{5}{3}x - \frac{1}{6}x^2$ and the equation of the line is $y = 4 - \frac{1}{3}x$.



(*a*) Find algebraically the area of wire mesh required for this part of the rockface. **(5)**

(*b*) To help secure the wire mesh, weights are attached to the mesh along the line $x = p$ so that the area of mesh is bisected.

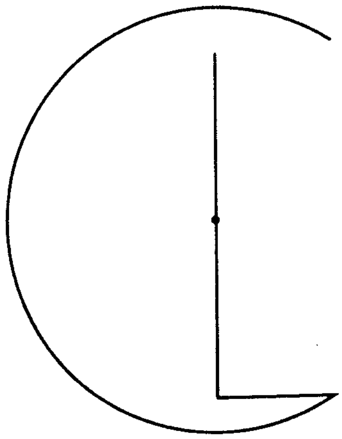
By using your answer to part (*a*), or otherwise, show that p satisfies the equation $$p^3 - 18p^2 + 432 = 0.$$ **(3)**

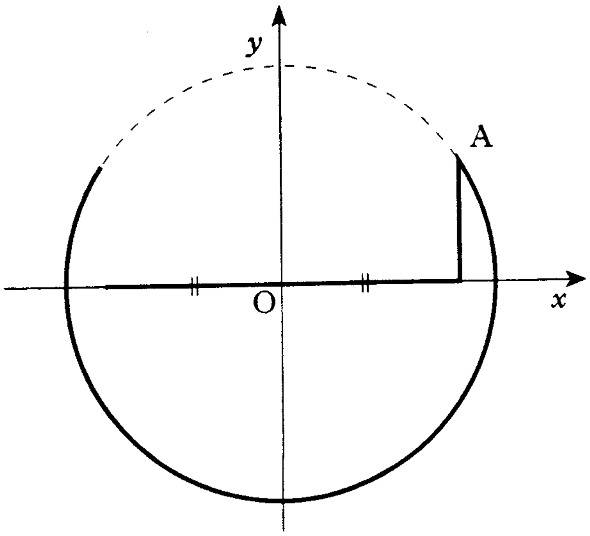
(*c*) (i) Verify that $p = 6$ is a solution of this equation.

(ii) Find algebraically the other two solutions of this equation.

(iii) Explain why $p = 6$ is the only valid solution to this problem. **(5)**

11. Linktown Church is considering designs for a logo for their parish magazine. The "C" is part of a circle and the centre of the circle is the mid-point of the vertical arm of the "L". Since the "L" is clearly smaller than the "C", the designer wishes to ensure that the total length of the arms of the "L" is as long as possible.





The designer decides to call the point where the "L" and "C" meet A and chooses to draw coordinate axes so that A is in the first quadrant. With axes as shown, the equation of the circle is $x^2 + y^2 = 20$.

(*a*) If A has coordinates (x,y), show that the total length T of the arms of the "L" is given by $T = 2x + \sqrt{20 - x^2}$. **(1)**

(*b*) Show that for a stationary value of T, x satisfies the equation

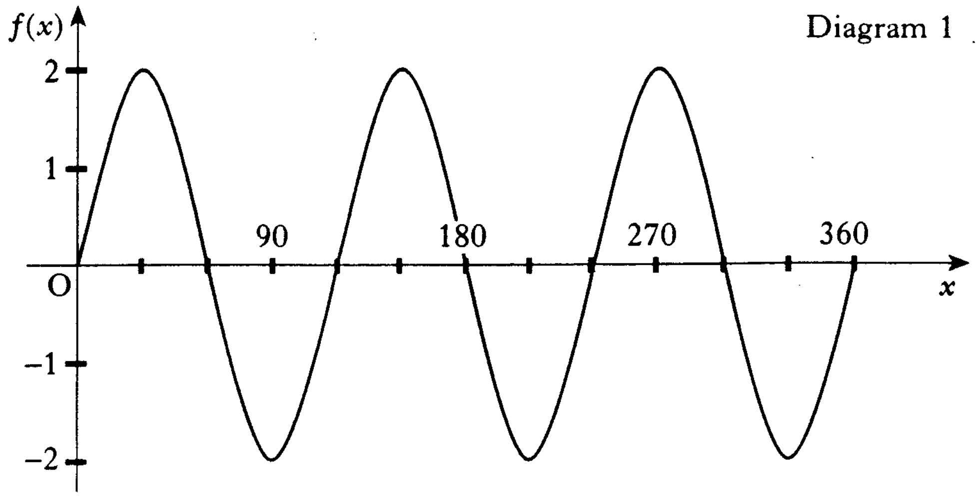
$$x = 2\sqrt{20 - x^2}.$$ **(5)**

(*c*) By squaring both sides, solve this equation.
Hence find the greatest length of the arms of the "L". **(3)**

[*END OF QUESTION PAPER*]

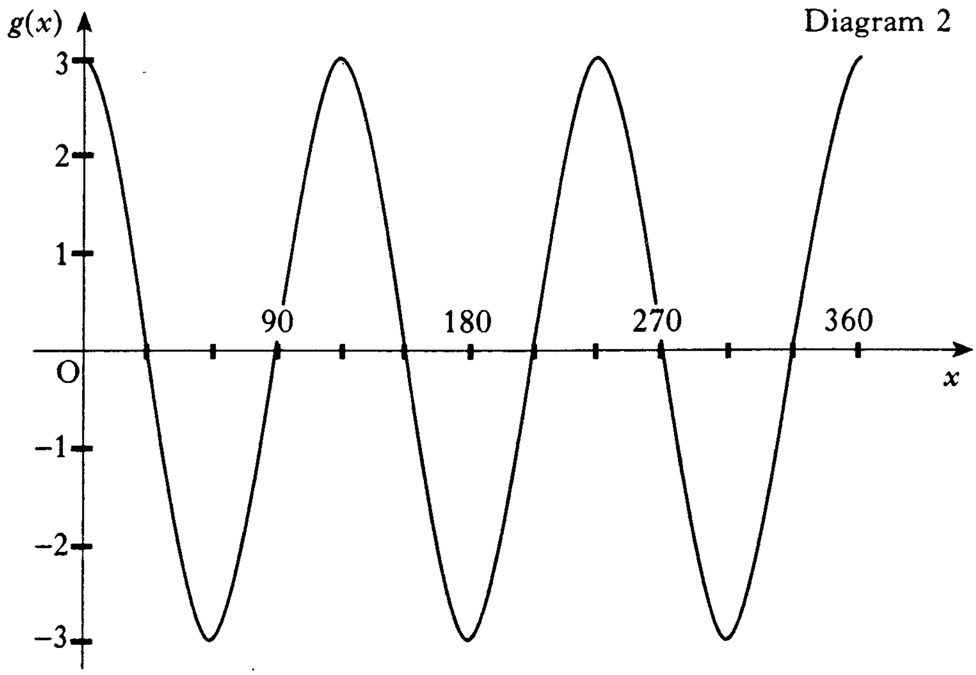
4. (*a*) (i) Diagram 1 shows part of the graph of the function f defined by $f(x) = b \sin ax°$, where a and b are constants.

Write down the values of a and b.



(ii) Diagram 2 shows part of the graph of the function g defined by $g(x) = d \cos cx°$, where c and d are constants.

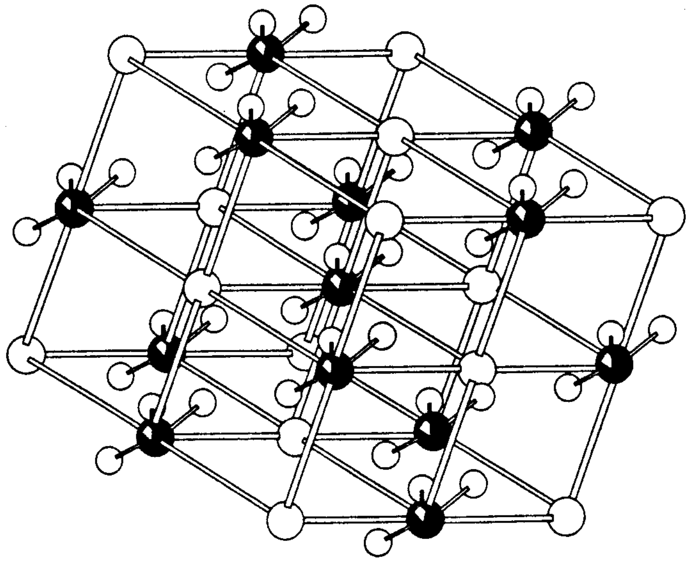
Write down the values of c and d. **(4)**



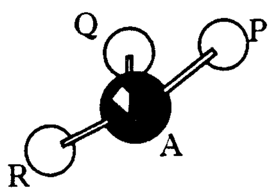
(*b*) The function h is defined by $h(x) = f(x) + g(x)$.

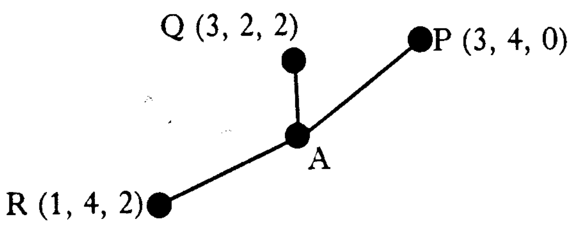
Show that $h(x)$ can be expressed in terms of a single trigonometric function of the form $q \sin(px + r)°$ and find the values of p, q and r. **(5)**

5. The diagram shows the rhombohedral crystal lattice of calcium carbonate.



The three oxygen atoms P, Q and R around the carbon atom A have coordinates as shown.





(*a*) Calculate the size of angle PQR. **(4)**

(*b*) M is the midpoint of QR and T is the point which divides PM in the ratio 2:1.

(i) Find the coordinates of T.

(ii) Show that P, Q and R are equidistant from T. **(6)**

(*c*) The coordinates of A are (2, 3, 1).

(i) Show that P, Q and R are also equidistant from A.

(ii) Explain why T, and not A, is the centre of the circle through P, Q and R. **(2)**