

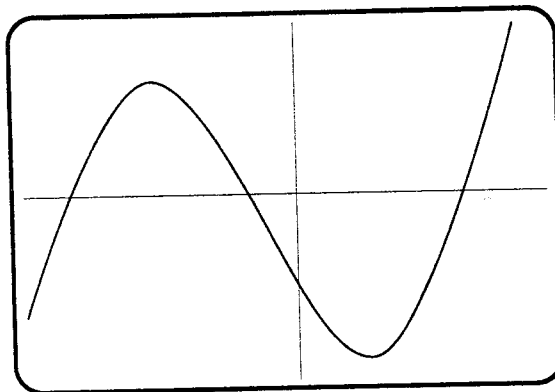
All questions should be attempted

Marks

1. The diagram shows part of the graph of the curve with equation

$$f(x) = x^3 + x^2 - 16x - 16.$$

- Factorise $f(x)$.
- Write down the coordinates of the four points where the curve crosses the x and y axes.
- Find the coordinates of the turning points and justify their nature.



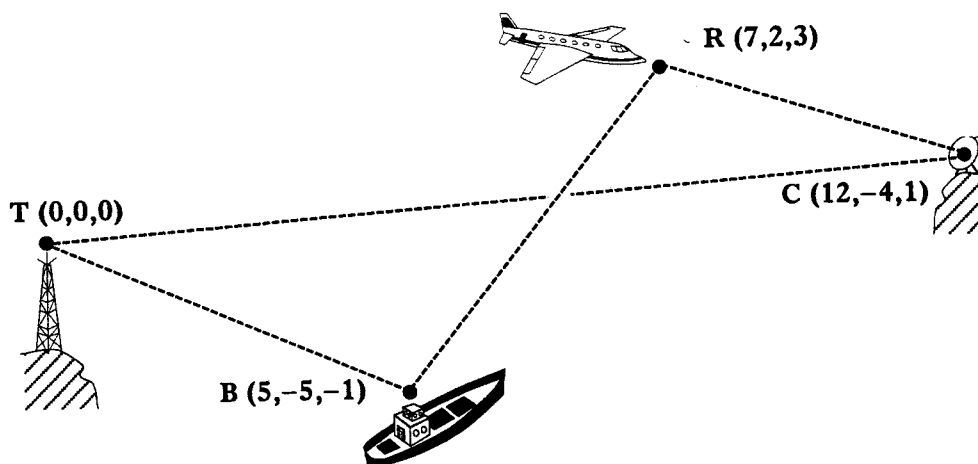
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(11)

2. Relative to a suitable set of coordinate axes with a scale of 1 unit to 2 kilometres, the positions of a transmitter mast, ship, aircraft and satellite dish are shown in the diagram below.



The top T of the transmitter mast is the origin, the bridge B on the ship is the point $(5, -5, -1)$, the centre C of the dish on the top of a mountain is the point $(12, -4, 1)$ and the reflector R on the aircraft is the point $(7, 2, 3)$.

- Find the distance in kilometres from the bridge of the ship to the reflector on the aircraft.
- Three minutes earlier, the aircraft was at the point $M(-2, 4, 8.5)$. Find the speed of the aircraft in kilometres per hour.
- Prove that the direction of the beam TC is perpendicular to the direction of the beam BR.
- Calculate the size of angle TCR.

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3. Biologists calculate that, when the concentration of a particular chemical in a sea loch reaches 5 milligrams per litre (mg/l), the level of pollution endangers the life of the fish.

A factory wishes to release waste containing this chemical into the loch. It is claimed that the discharge will not endanger the fish.

The Local Authority is supplied with the following information:

1. The loch contains none of this chemical at present.
2. The factory manager has applied to discharge waste once per week which will result in an increase in concentration of 2.5 mg/l of the chemical in the loch.
3. The natural tidal action will remove 40% of the chemical from the loch every week.

- (a) Show that this level of discharge would result in fish being endangered. 3

When this result is announced, the company agrees to install a cleaning process that reduces the concentration of chemical released into the loch by 30%.

- (b) Show the calculations you would use to check this revised application.
Should the Local Authority grant permission? 5

(8)

4. (a) For a particular radioactive substance, the mass m (in grams) at time t (in years) is given by

$$m = m_0 e^{-0.02t}$$

where m_0 is the original mass.

If the original mass is 500 grams, find the mass after 10 years. 2

- (b) The half-life of any material is the time taken for half of the mass to decay.

Find the half-life of this substance. 3

- (c) Illustrate **all** of the above information on a graph. 3

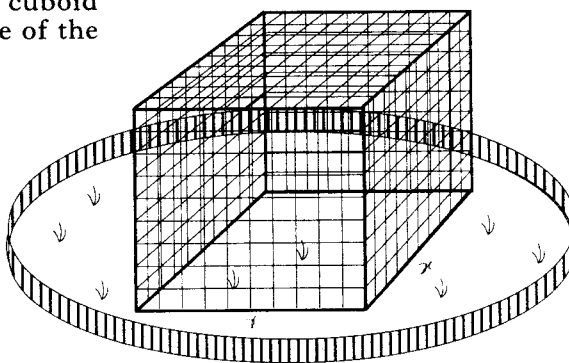
(8)

5. The owners of a zoo intend to build a new aviary in the shape of a cuboid with a square floor. The volume of the aviary will be 500 m^3 .

- (a) If x metres is the length of one edge of the floor, show that the area A square metres of netting required is given by

$$A = x^2 + \frac{2000}{x}$$

- (b) Find the dimensions of the aviary to ensure that the cost of netting is minimised.



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6. The *vector product*, $\mathbf{a} \times \mathbf{b}$, of $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ is defined by

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}.$$

Example

When $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$

then $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} 2 \times 2 - 3 \times 0 \\ 3 \times (-1) - 1 \times 2 \\ 1 \times 0 - 2 \times (-1) \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix}.$

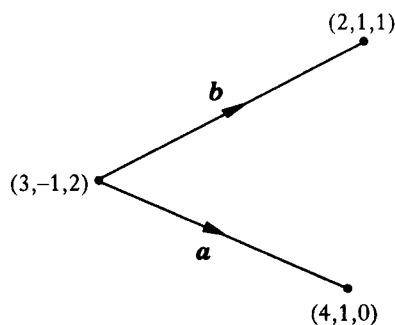
- (a) If \mathbf{a} and \mathbf{b} are as shown in the diagram and $\mathbf{c} = \mathbf{a} \times \mathbf{b}$, evaluate \mathbf{c} .

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- (b) By considering $\mathbf{a} \cdot \mathbf{c}$ and $\mathbf{b} \cdot \mathbf{c}$, what can be concluded about \mathbf{c} ?

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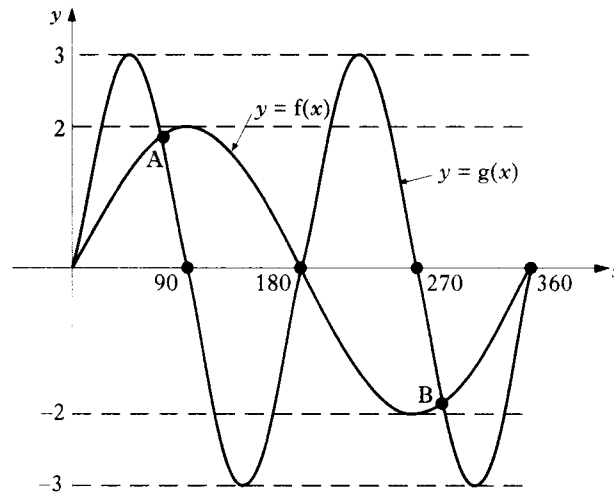
(7)



Marks

7. (a) Solve the equation $3\sin 2x^\circ = 2\sin x^\circ$ for $0 \leq x \leq 360$. 4
- (b) The diagram below shows parts of the graphs of sine functions f and g .
State expressions for $f(x)$ and $g(x)$. 1
- (c) Use your answers to part (a) to find the coordinates of A and B. 2
- (d) Hence state the values of x in the interval $0 \leq x \leq 360$ for which $3\sin 2x^\circ < 2\sin x^\circ$. 3

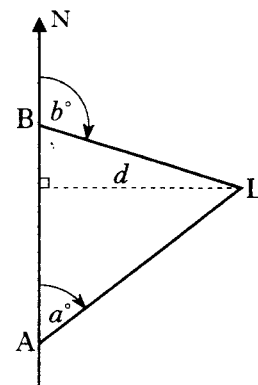
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8. A ship is sailing due north at a constant speed.

When at position A, a lighthouse L is observed on a bearing of a° . One hour later, when the ship is at position B, the lighthouse is on a bearing of b° .

The shortest distance between the ship and the lighthouse during this hour was d miles.



- (a) Prove that $AB = \frac{d}{\tan a^\circ} - \frac{d}{\tan b^\circ}$. 2
- (b) Hence prove that $AB = \frac{d \sin(b - a)^\circ}{\sin a^\circ \sin b^\circ}$. 3
- (c) Calculate the shortest distance from the ship to the lighthouse when the bearings a° and b° are 060° and 135° respectively and the constant speed of the ship is 14 miles per hour. 3

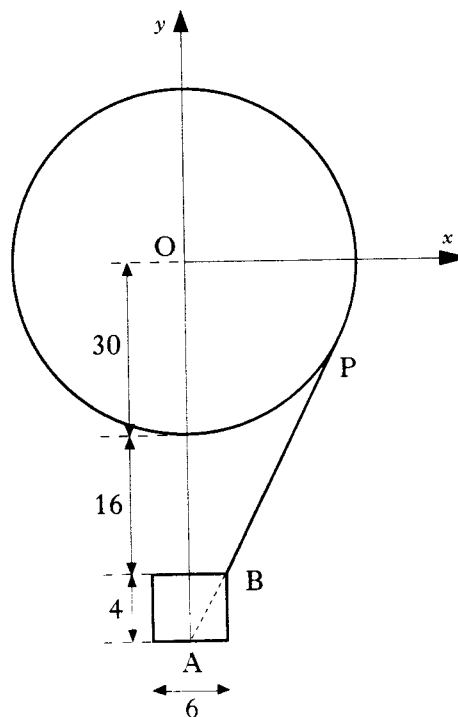
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9.



A spherical hot-air balloon has radius 30 feet. Cables join the balloon to the gondola which is cylindrical with diameter 6 feet and height 4 feet. The top of the gondola is 16 feet below the bottom of the balloon.

Coordinate axes are chosen as shown in the diagram. One of the cables is represented by PB and PBA is a straight line.



- Find the equation of the cable PB.
- State the equation of the circle representing the balloon.
- Prove that this cable is a tangent to the balloon and find the coordinates of the point P.

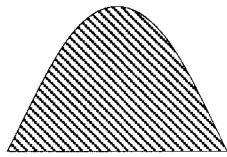
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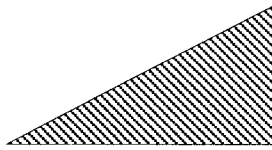
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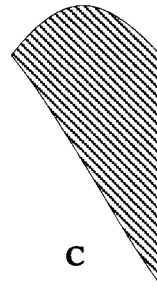
10. An artist has been asked to design a window made from pieces of coloured glass with different shapes. To preserve a balance of colour, each shape must have the **same** area. Three of the shapes used are drawn below.



A



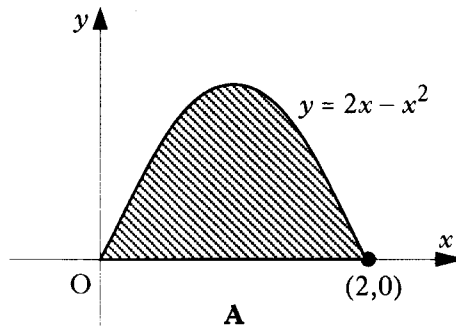
B



C

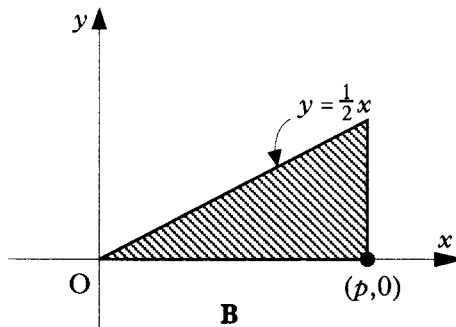
Relative to x, y -axes, the shapes are positioned as shown below. The artist drew the curves accurately by using the equation(s) shown in each diagram.

- (a) Find the area shaded under $y = 2x - x^2$.



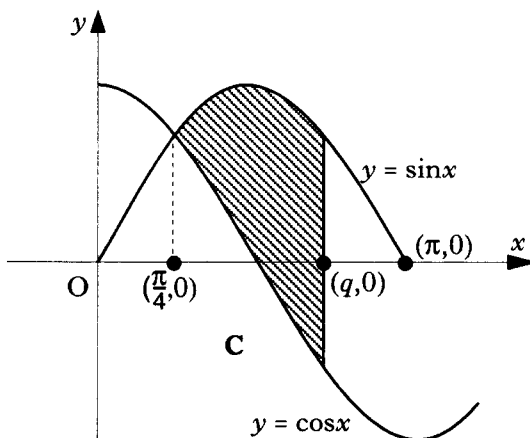
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- (b) Use the area found in part (a) to find the value of p .



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- (c) Prove that q satisfies the equation $\cos q + \sin q = 0.081$ and hence find the value of q to 2 significant figures.

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(16)

[END OF QUESTION PAPER]