SCOTTISH CERTIFICATE OF EDUCATION 1992

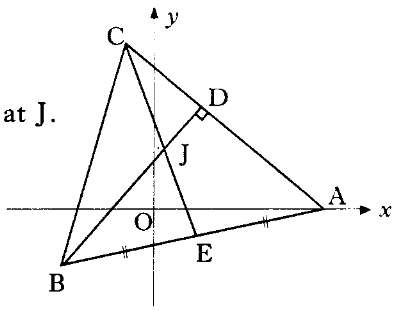
MONDAY, 11 MAY
9.30 AM – 11.30 AM

MATHEMATICS HIGHER GRADE
Paper I

All questions should be attempted *Marks*

1. Find the equation of the tangent to the curve with equation $y = 5x^3 - 6x^2$ at the point where $x = 1$. **(4)**

2. In the diagram, A is the point (7, 0), B is (−3, −2) and C (−1, 8).
The median CE and the altitude BD intersect at J.
(*a*) Find the equations of CE and BD.
(*b*) Find the coordinates of J. **(8)**



3. Find k if $x - 2$ is a factor of $x^3 + kx^2 - 4x - 12$. **(3)**

4. A curve for which $\dfrac{dy}{dx} = 3x^2 + 1$ passes through the point (−1, 2).
Express y in terms of x. **(4)**

5. Find, correct to one decimal place, the value of x between 180 and 270 which satisfies the equation $3\cos(2x - 40)° - 1 = 0$. **(5)**

6. On a suitable set of real numbers, functions f and g are defined by

$$f(x) = \frac{1}{x+2} \quad \text{and} \quad g(x) = \frac{1}{x} - 2.$$

Find $f(g(x))$ in its simplest form. **(3)**

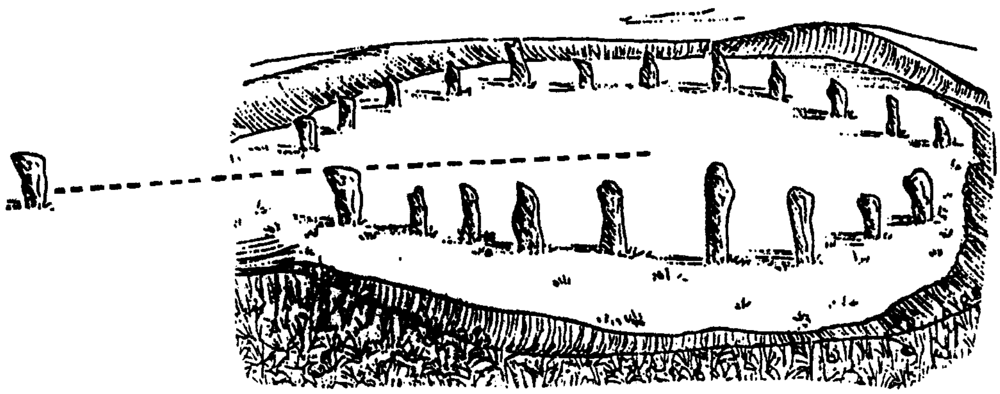
7. (*a*) Express $\sin x° - 3\cos x°$ in the form $k\sin(x - \alpha)°$ where $k > 0$ and $0 \leqslant \alpha < 360$. Find the values of k and α.

(*b*) Find the maximum value of $5 + \sin x° - 3\cos x°$ and state a value of x for which this maximum occurs. **(6)**

8. Evaluate $\displaystyle\int_1^9 \frac{x+1}{\sqrt{x}}\,dx$. **(5)**

Marks

9.



An ancient Stone Circle has a processional pathway from the Heelstone to the centre of the Stone Circle. In the picture above, the Heelstone is on the left and the dotted line represents the processional pathway.

With suitable axes and using the Heelstone as the origin, the equation of the Stone Circle is $x^2 + y^2 - 8x - 6y + 21 = 0$.

Given that 1 unit represents 15 metres, calculate the distance in metres from the Heelstone to the nearest point on the edge of the Circle. **(5)**

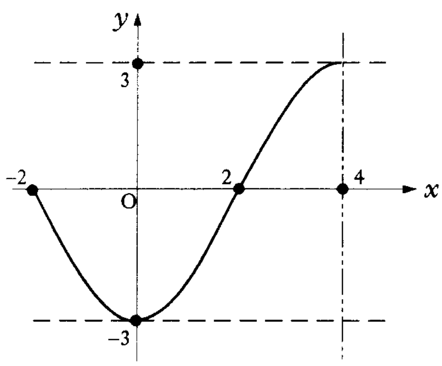
10. The sketch shows the graph of $y = f(x)$ for $-2 \leqslant x \leqslant 4$.

The function $g(x)$ has the line $x = 4$ as an axis of symmetry and $g(x) = f(x)$ for $-2 \leqslant x \leqslant 4$.

On separate sketches, indicate

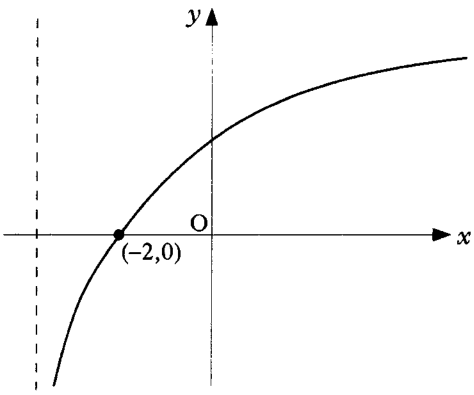
(*a*) $y = g(x)$ for $-2 \leqslant x \leqslant 10$

(*b*) $y = -2g(x)$ for $0 \leqslant x \leqslant 8$. **(4)**



11. Differentiate $2x^{\frac{3}{2}} + \sin^2 x$ with respect to x. **(4)**

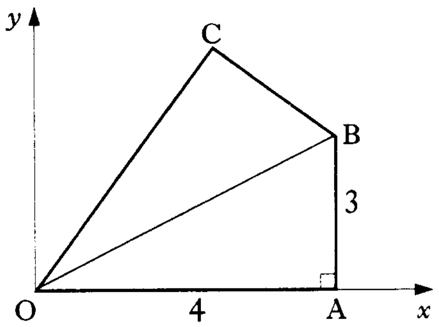
12. An incomplete sketch (not drawn to scale) of the graph of $y = \log_{10}(x + a)$ is shown. Find the value of a. **(2)**



Marks

13. The diagram shows a kite OABC. A is the point (4,0) and B is the point (4,3).

Calculate the gradient of OC correct to two decimal places.

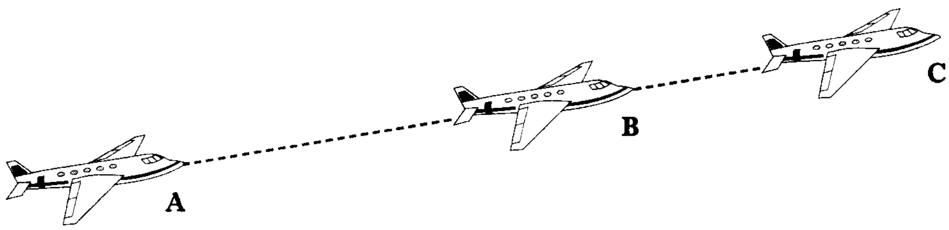


(3)

14. (*a*) Evaluate $\int_0^{\pi/2} \cos 2x \, dx$.

(*b*) Draw a sketch and explain your answer. **(5)**

15.



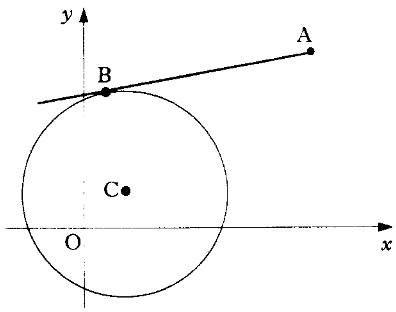
An aircraft flying at a constant speed on a straight flight path takes 2 minutes to fly from A to B and 1 minute to fly from B to C. Relative to a suitable set of axes, A is the point $(-1,3,4)$ and B is the point $(3, 1, -2)$. Find the coordinates of the point C. **(3)**

16. AB is a tangent at B to the circle with centre C and equation

$(x - 2)^2 + (y - 2)^2 = 25.$

The point A has coordinates (10,8).

Find the area of triangle ABC.



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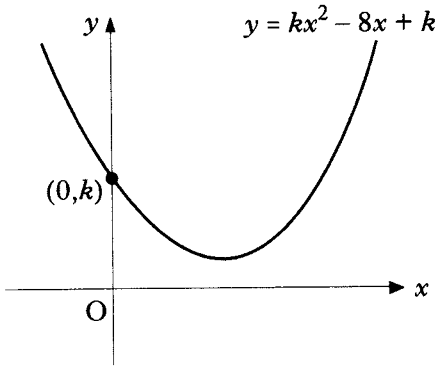
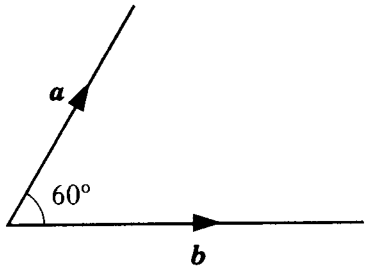
17. Calculate the least positive integer value of k so that the graph of $y = kx^2 - 8x + k$ does not cut or touch the x-axis. **(4)**



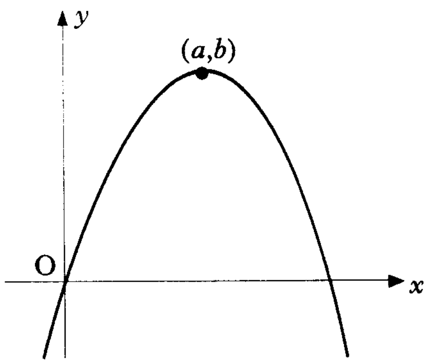
18. The diagram shows representatives of two vectors, $\boldsymbol{a}$ and $\boldsymbol{b}$, inclined at an angle of 60°.

If $|\boldsymbol{a}| = 2$ and $|\boldsymbol{b}| = 3$, evaluate $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b})$. **(3)**



19. The line with equation $y = x$ is a tangent at the origin to the parabola with equation $y = f(x)$. The parabola has a maximum turning point at (a, b).

Sketch the graph of $y = f'(x)$. **(4)**



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