

All questions should be attempted.

Marks

1. Find  $p$  if  $(x + 3)$  is a factor of  $x^3 - x^2 + px + 15$ .

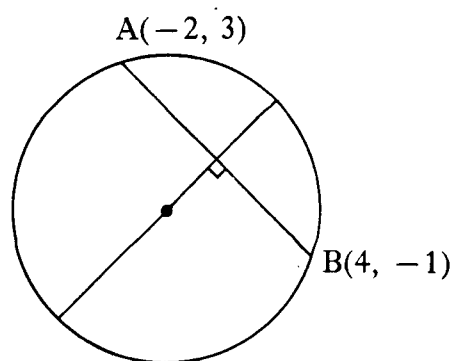
(3)

2. Find the equation of the tangent to the curve  $y = 4x^3 - 2$  at the point where  $x = -1$ .

(4)

3. A circle passes through  $A(-2, 3)$  and  $B(4, -1)$ .

Find the equation of the diameter which is perpendicular to the chord  $AB$ .



(4)

4. Show that  $P(2, 2, 3)$ ,  $Q(4, 4, 1)$  and  $R(5, 5, 0)$  are collinear and find the ratio in which  $Q$  divides  $PR$ .

(4)

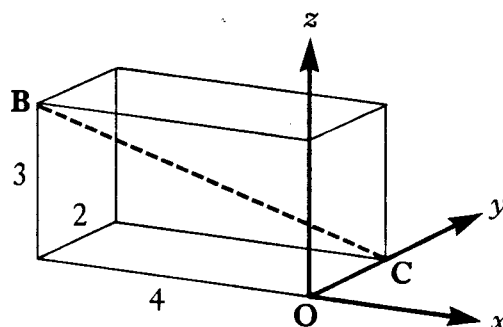
5. A cuboidal crystal is placed relative to the coordinate axes as shown opposite.

(a) Write down  $\overrightarrow{BC}$  in component form.

1

(b) Calculate  $|\overrightarrow{BC}|$ .

1



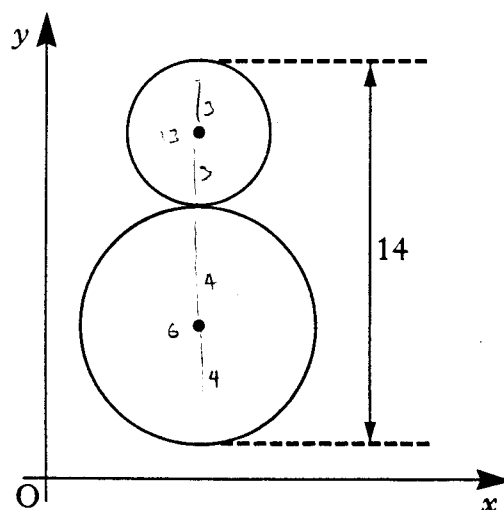
(2)

6. Evaluate  $\int_1^2 (3x^2 + 4) dx$  and draw a sketch to illustrate the area represented by this integral.

(5)

7. A bakery firm makes gingerbread men each 14 cm high with a circular “head” and “body”.

The equation of the “body” is  $x^2 + y^2 - 10x - 12y + 45 = 0$  and the line of centres is parallel to the  $y$ -axis. Find the equation of the “head”.



(5)

8. For all points on a curve  $y = f(x)$ ,  $f'(x) = 1 - 2x$ .  
If the curve passes through the point  $(2, 1)$ , find the equation of the curve. (3)

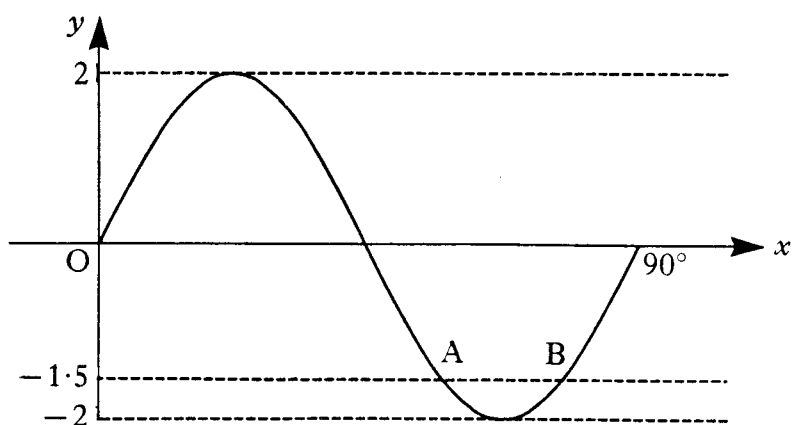
9. Given that  $\cos D = \frac{2}{\sqrt{5}}$  and  $0 < D < \frac{\pi}{2}$ , find the exact values of  $\sin D$  and  $\cos 2D$ . (4)

10. The diagram opposite shows the graph of a sine function from  $0^\circ$  to  $90^\circ$ .

(a) State the equation of the graph.

(b) The line with equation  $y = -1.5$  intersects the curve at A and B.

Find the coordinates of A and B.



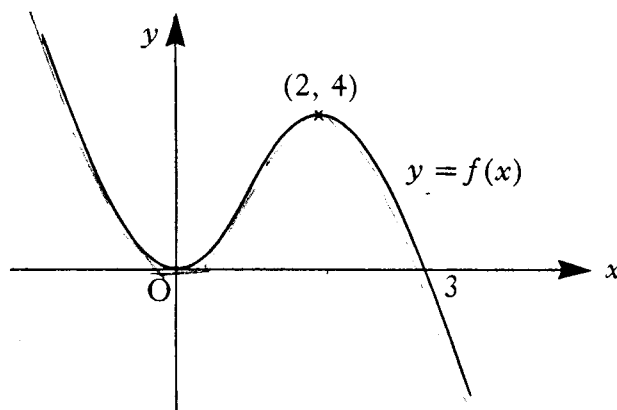
2

3

(5)

11. The diagram opposite shows a sketch of the cubic function  $f$  with stationary points at  $(0, 0)$  and  $(2, 4)$ .

Sketch the graph of the derived function  $f'$ .



(3)

12. The vector  $ai + bj + k$  is perpendicular to both the vectors  $i - j + k$  and  $-2i + j + k$ .

Find the values of  $a$  and  $b$ .

(3)

13. (a) Find the coordinates of the points of intersection of the curves with equations  $y = 2x^2$  and  $y = 4 - 2x^2$ .

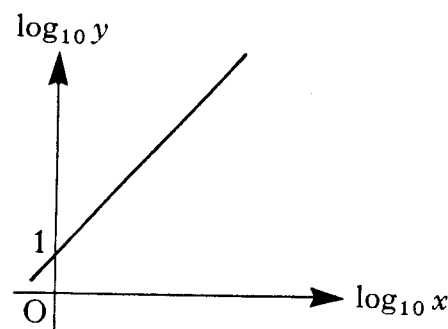
2

(b) Find the area completely enclosed between these two curves.

3

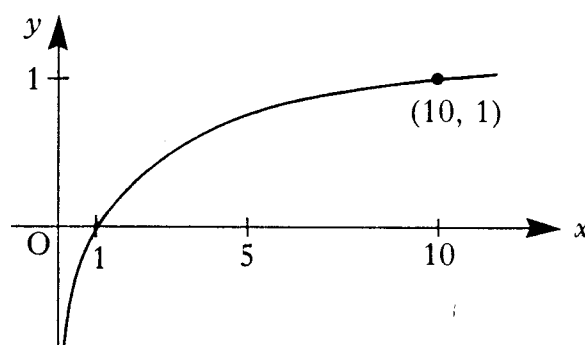
(5)

14. As shown in the diagram opposite, a set of experimental results gives a straight line graph when  $\log_{10} y$  is plotted against  $\log_{10} x$ . The straight line passes through (0, 1) and has a gradient of 2. Express  $y$  in terms of  $x$ .



(6)

15. Solve the equation  $2\cos^2 x = \frac{1}{2}$  for  $0 \leq x \leq \pi$ . (3)
16. For what values of  $x$  is the function  $f(x) = \frac{1}{3}x^3 - 2x^2 - 5x - 4$  increasing? (5)
17. Make a copy of this graph of  $y = \log_{10} x$ . On your copy, sketch the graph of  $y = \log_{10}(x - 2)$ .



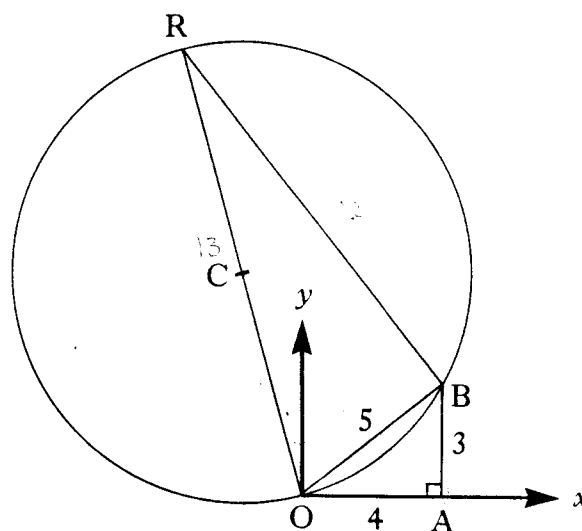
(3)

18. Show that the roots of the equation  $(k - 2)x^2 - (3k - 2)x + 2k = 0$  are real. (4)
19. If  $f(x) = \cos^2 x - \frac{2}{3x^2}$ , find  $f'(x)$ . (4)

20. The right-angled triangle OAB with sides of length 3 cm, 4 cm and 5 cm is placed with one vertex at the origin O as shown in the diagram.

A circle centre C and diameter RO of length 13 cm is drawn and passes through O and B.

What is the gradient of the line RO?



(5)

[END OF QUESTION PAPER]

**All questions should be attempted.**

*Marks*

1. A function  $f$  is defined by the formula  $f(x) = (x - 1)^2(x + 2)$  where  $x \in \mathbf{R}$ .
- (a) Find the coordinates of the points where the curve with equation  $y = f(x)$  crosses the  $x$ - and  $y$ -axes. 3
  - (b) Find the stationary points of this curve  $y = f(x)$  and determine their nature. 7
  - (c) Sketch the curve  $y = f(x)$ . 2
- (12)**
2. P, Q and R have coordinates  $(1, -2)$ ,  $(6, 3)$  and  $(9, 14)$  respectively and are three vertices of a kite PQRS.
- (a) Find the equations of the diagonals of this kite and the coordinates of the point where they intersect. 7
  - (b) Find the coordinates of the fourth vertex S. 2
- (9)**
3. The extract below is taken from the "OIL RIG NEWS".

### **RARE ILLNESS STRIKES RIG**

#### **Storm prevents delivery of medicine**

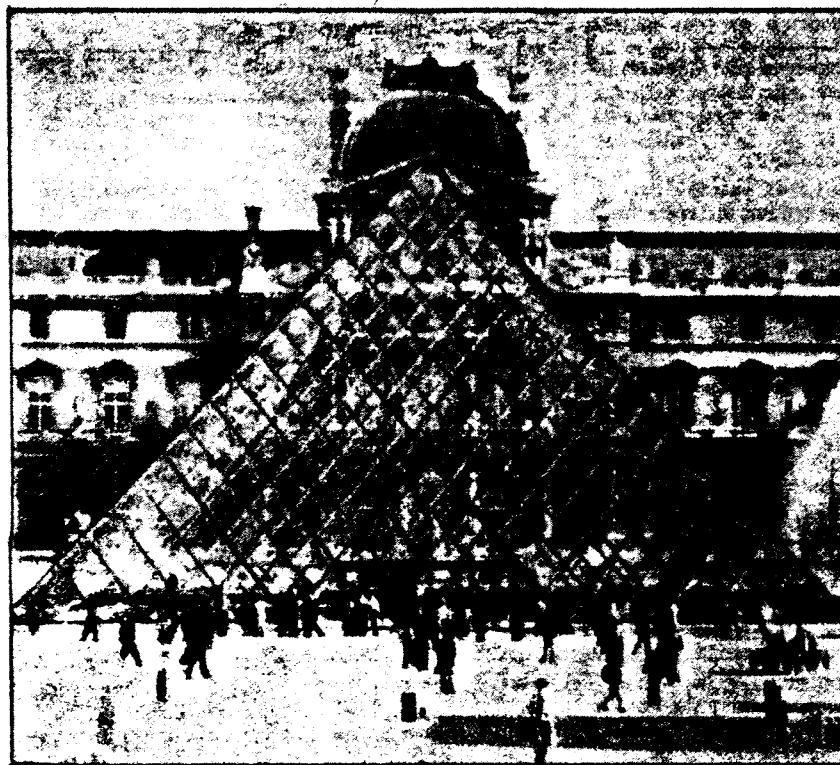
By noon on Tuesday 20 December 1988  
50 of our oil rig personnel were laid low  
by a mystery illness.

Our resident medical officer is expressing  
concern because the number of personnel  
affected is increasing each day by 8% of  
the previous day's total.

- (a) If the daily rate of increase remained at 8% of the previous day's total, how many personnel were affected by noon on Sunday 25 December 1988? 3
  - (b) An improvement in the weather conditions allowed a team of medics to fly out to the rig on the morning of Tuesday 27 December 1988.  
At noon on that Tuesday, all personnel were inoculated and no new cases of the illness arose. Within the next 24 hours, 21% of those who had been affected had recovered.  
If the daily rate of recovery of 21% of the previous day's total was maintained, how many personnel were still affected by the illness at noon on Saturday 31 December 1988? 5
- (8)**

**[Turn over**

4. This is an extract from a newspaper article concerning the new entrance to the Louvre Museum.

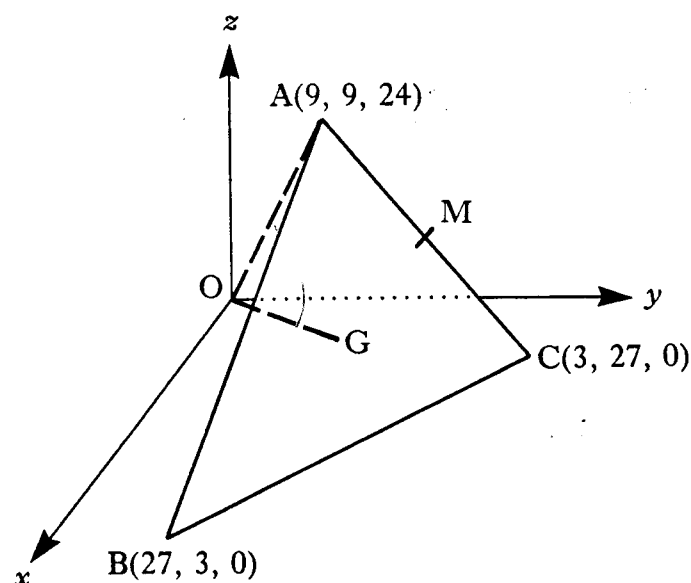


President Mitterrand yesterday unveiled a controversial giant glass pyramid destined to be the new entrance to the Louvre Museum in Paris.

- (a) Relative to the mutually perpendicular axes  $Ox$ ,  $Oy$  and  $Oz$ , the front face of this pyramid is represented by triangle  $ABC$ , where  $A$  is the point with coordinates  $(9, 9, 24)$ ,  $B$  is the point  $(27, 3, 0)$ ,  $C$  is the point  $(3, 27, 0)$  and  $M$  is the mid-point of  $AC$ , as shown in the diagram below.

Find the coordinates of  $G$  which divides  $BM$  in the ratio  $2:1$ .

3



- (b) Support girders are to be erected from  $O$  to  $G$  and from  $O$  to  $A$ . Calculate the size of the angle between the girders.

5

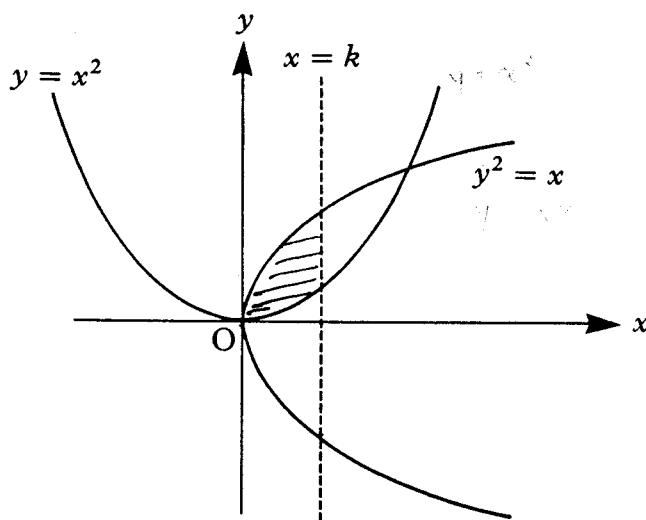
(8)

Marks

5. (a) Show that  $2\cos(x + 30)^\circ - \sin x^\circ$  can be written as  $\sqrt{3}\cos x^\circ - 2\sin x^\circ$ . 3
- (b) Express  $\sqrt{3}\cos x^\circ - 2\sin x^\circ$  in the form  $k\cos(x + \alpha)^\circ$  where  $k > 0$  and  $0 < \alpha < 360$  and find the values of  $k$  and  $\alpha$ . 4
- (c) Hence, or otherwise, solve the equation  $2\cos(x + 30)^\circ = \sin x^\circ + 1$ ,  $0 \leq x \leq 360$ . 3
- (10)

6. (a) The function  $f$  is defined by  $f(x) = x^3 - 2x^2 - 5x + 6$ .  
The function  $g$  is defined by  $g(x) = x - 1$ .  
Show that  $f(g(x)) = x^3 - 5x^2 + 2x + 8$ . 4
- (b) Factorise fully  $f(g(x))$ . 3
- (c) The function  $k$  is such that  $k(x) = \frac{1}{f(g(x))}$ .  
For what values of  $x$  is the function  $k$  not defined? 2
- (9)

7. The diagram shows two curves with equations  $y = x^2$  and  $y^2 = x$ .



The area completely enclosed between these two curves is divided in half by the line with equation  $x = k$ .

- (a) Represent these two equal areas by two separate integrals each involving  $k$ . 6
- (b) Equate the integrals and show that  $k$  is given by the equation  $2k^3 - 4k^{\frac{3}{2}} + 1 = 0$ . 4
- (c) Use the substitution  $p^2$  for  $k^3$  to find the value of  $k$ . 4
- (14)

[Turn over

8. A sports club awards trophies in the form of paperweights bearing the club crest.

Diagram 1 shows the front view of one of these paperweights.

Each is made from two different types of glass. The two circles are concentric and the base line is a tangent to the inner circle.

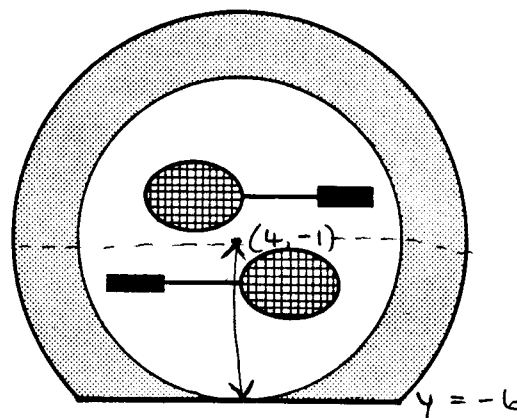


Diagram 1

- (a) Relative to  $x, y$  coordinate axes, the equation of the outer circle is

$$x^2 + y^2 - 8x + 2y - 19 = 0$$

and the equation of the base line is  $y = -6$ .

Show that the equation of the inner circle is  $x^2 + y^2 - 8x + 2y - 8 = 0$ .

4

- (b) An alternative form of the paperweight is made by cutting off a piece of glass from the original design along a second line with equation  $3x - 4y + 9 = 0$  as shown in Diagram 2.

Show that this line is a tangent to the inner circle and state the coordinates of the point of contact.

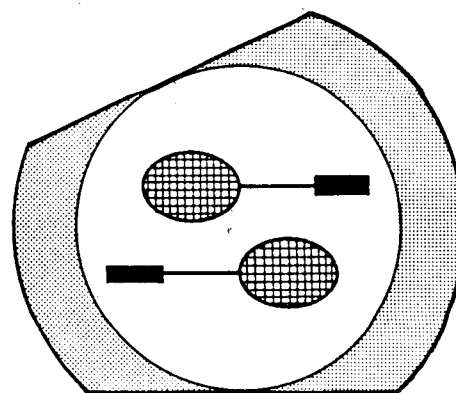


Diagram 2

7

(11)

9. Polynomial equations often have roots which are not whole numbers.

One method of estimating the roots of such equations is to make repeated use of the following:

If  $x = p$  is an estimate of a root of the equation  $f(x) = 0$ , then  $x = q$  will be a closer estimate where

$$q = p - \frac{f(p)}{f'(p)}$$

### EXAMPLE

One of the roots of the equation  $x^2 - 2x - 5 = 0$  is known to lie between 3 and 4. The above rule can be used to estimate this root to one decimal place as follows:

$$f(x) = x^2 - 2x - 5 \quad \text{so } f'(x) = 2x - 2$$

Choose  $p = 3$  (1st estimate)

$$\text{then } q = 3 - \frac{f(3)}{f'(3)} = 3 - \frac{(-2)}{4} = 3.5$$

Hence  $x = 3.5$  is a closer estimate to the root than  $x = 3$

Choose  $p = 3.5$  (2nd estimate)

$$\text{then } q = 3.5 - \frac{f(3.5)}{f'(3.5)} = 3.5 - \frac{0.25}{5} = 3.45$$

Hence  $x = 3.45$  is a closer estimate to the root than  $x = 3.5$

Choose  $p = 3.45$  (3rd estimate)

$$\text{then } q = 3.45 - \frac{f(3.45)}{f'(3.45)} = 3.45 - \frac{0.0025}{4.9} = 3.449$$

Hence  $x = 3.449$  is a closer estimate to the root than  $x = 3.45$

**CONCLUSION** The root, correct to one decimal place, is  $x = 3.4$

- (a) Show that the equation  $x^3 - 2x^2 + 6x - 4 = 0$  has a root between 0 and 1. 3  
 (b) Use the method described above to find this root correct to one decimal place. 6

(9)

[Turn over



10. The Water Board of a local authority discovered it was able to represent the approximate amount of water  $W(t)$ , in millions of gallons, stored in a reservoir  $t$  months after 1st May 1988 by the formula

$$W(t) = 1.1 - \sin \frac{\pi t}{6}$$

The board then predicted that under normal conditions this formula would apply for three years.

- (a) Draw and label sketches of the graphs of  $y = \sin \frac{\pi t}{6}$  and  $y = -\sin \frac{\pi t}{6}$ , for  $0 \leq t \leq 36$  on the same diagram. 4
- (b) On a separate diagram and using the same scale on the  $t$ -axis as you used in part (a), draw a sketch of the graph of  $W(t) = 1.1 - \sin \frac{\pi t}{6}$ . 3
- (c) On the 1st April 1990 a serious fire required an extra  $\frac{1}{4}$  million gallons of water from the reservoir to bring the fire under control. Assuming that the previous trend continues from the new lower level, when will the reservoir run dry if water rationing is not imposed? 3

(10)

[END OF QUESTION PAPER]