Duncanrig Secondary School – Mathematics Department

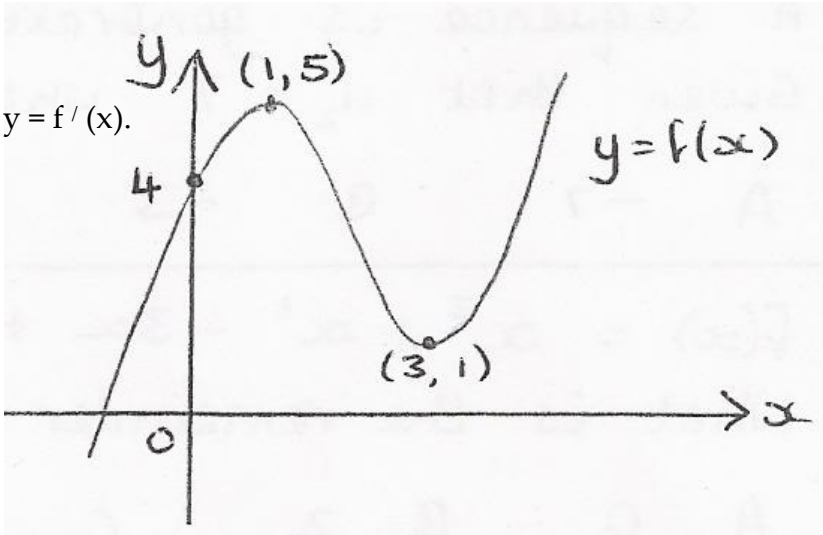
HIGHER MATHS HOME EXERCISE 16

1. A curve has equation $y = \frac{1}{2}(2x^3 - 9x^2 + 12x)$.

(a) Find algebraically the coordinates of the stationary points.
(b) Determine the nature of the stationary points.

2. Vector **p** is given by $3\mathbf{i} + \mathbf{j} - \mathbf{k}$ and **q** is $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$. What are the components of $2\mathbf{p} - \mathbf{q}$?

3. Shown is a sketch of the graph of $y = f(x)$.
Sketch the graph of the derived function $y = f'(x)$.



4. K and L have position vectors $\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$ respectively. What is the magnitude of $\overrightarrow{KL}$?

5. Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$ are defined on the set of real numbers.
(a) Find $h(x)$ where $h(x) = g(f(x))$.
(b) (i) Write down the coordinates of the minimum turning point of $y = h(x)$.
(ii) Hence state the range of the function h.

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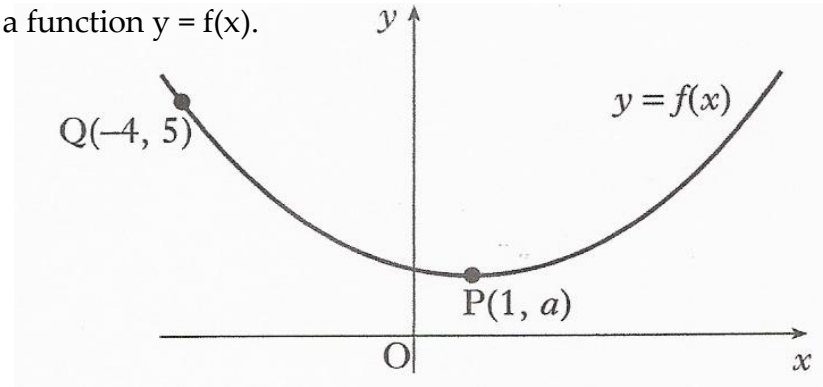
6. The diagram shows the graph of a function y = f(x).

Copy the diagram and on it sketch the graphs of:

(a) $y = f(x - 4)$

(b) $y = 2 + f(x - 4)$.



7. (a) Calculate the limit as $n \to \infty$ of the sequence defined by

$$u_{n+1} = 0{\cdot}9\,u_n + 10,\ u_0 = 1.$$

(b) Determine the least value of n for which u_n is greater than half of this limit and the corresponding value of u_n.

8. Given $f(x) = 3x^2(2x - 1)$, find $f'(-1)$.

9. Show that the points $P(-6,-1)$, $Q(0,2)$ and $R(8,6)$ are collinear.

10. The vectors $\mathbf{u} = \begin{pmatrix} k \\ -1 \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 0 \\ 4 \\ k \end{pmatrix}$ are perpendicular. What is the value of k?

11. For what values of x is $f(x) = x^3 - 12x + 2$ decreasing?

12. If $3\log_2 t - \log_2 5 = 3$, find the value of t.

13. What is the value of $\dfrac{\log_2 32}{\log_2 8}$?

14. Solve the equation $\log_4(5 - x) - \log_4(3 - x) = 2$, $x < 3$.